HUMBOLDT-UNIVERSITÄT ZU BERLIN

Count on the Brain: Using EEG Oscillations and Eye Movements to Disentangle Intelligent Problem-Solving in Math

Dissertation

zur Erlangung des akademischen Grades

Doktor rerum naturalium (Dr. rer. nat.)

im Fach Psychologie

Eingereicht an der Lebenswissenschaftlichen Fakultät

der Humboldt-Universität zu Berlin

von

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Berlin, den 28.07.2015

Tag der Verteidigung: 14.12.2015

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DANKSAGUNG

Diese Arbeit ist nicht das Werk einer einzelnen Person und wäre ohne die Hilfe anderer nicht möglich gewesen. Daher möchte ich im Nachfolgenden allen Beteiligten meine aufrichtige Dankbarkeit zum Ausdruck bringen.

An erster Stelle möchte ich Prof. Dr. Elke van der Meer und Prof. Dr. Isabell Wartenburger meinen Dank aussprechen. Frau van der Meer, ich möchte Ihnen danken, dass Sie mich in die Welt des wissenschaftlichen Arbeitens eingeführt haben, dass ich jederzeit Ihren Rat erfragen konnte, für Ihre Unterstützung gerade dann, wenn drei Jahre schneller vorbei sind als man denkt, und vor allem für Ihre Freundlichkeit und Herzlichkeit. Vielen Dank, Isabell, für Deine wertvollen Hinweise und Ideen, für die Zeit, die Du Dir für mich und meine Arbeit genommen hast, und für die wertschätzenden und aufmunternden Worte.

Ich möchte außerdem den Mitarbeitern am Lehrstuhl für Kognitive Psychologie der Humboldt-Universität zu Berlin für die Unterstützung, den förderlichen wissenschaftlichen Austausch und die angenehme Arbeitsatmosphäre in den vergangenen Jahren danken. Ich danke insbesondere Christina Rügen, Susanne Holzinger, Sabine Schulz, Franziska Illner und Gesa Schaadt. Vielen Dank, Christina, für Deinen unermüdlichen Einsatz während der dicht gelegten Testtermine im Labor. Susanne, Dir danke ich für Deine Unterstützung in den Schulen zu Beginn meiner Promotion. Liebe Sabine, Dir möchte ich sagen, dass ich mich immer sehr wohl am Lehrstuhl gefühlt habe. Ich bedanke mich daher vor allem für Deine Gesellschaft, die wesentlich dazu beigetragen hat. Franziska, Dir danke ich als Kollegin und Freundin für die sprachliche Korrektur der Synopsis trotz anstrengender Klausurphase. Die schönste Erfahrung während meiner Promotionszeit war das gemeinsame Arbeiten mit Dir, Gesa. Ich möchte mich besonders für Deinen unfehlbaren Rat, Dein Planungstalent und nicht zuletzt für Deine Freundschaft bedanken, die mir an jedem Tag geholfen hat.

Des Weiteren gilt mein Dank Anna-Cara Hage und Linda Gerresheim für die Hilfe im Kampf mit den EEG-Artefakten und den anderen Studierenden des A-Teams – Leonie Bortfeldt, Meggie Danziger, Kristina Meyer und Phillipp Schoppmann – sowie Xenia Zimmermann für ihr Engagement während der Pilottestungen. Bei Keri Hartman und Jane Zagorski bedanke ich mich für die Sprachkorrektur der Manuskripte.

Meinen Dank möchte ich auch an das Management, die Mitarbeiter, das Kollegium und die assoziierten Wissenschaftler der Berlin School of Mind and Brain der Humboldt-Universität zu Berlin richten, die viel Zeit und Arbeit investieren, um Promovierenden wie mir ein facettenreiches und interessantes Doktorandenprogramm zu bieten. Ich bin sehr froh Teil von Kohorte 6 zu sein und bedanke mich bei meinen MitdoktorandInnen für das gemeinsame Durchleben von den Höhen und Tiefen einer Promotion und für den wissenschaftlichen und nicht-wissenschaftlichen Austausch.

Überdies wären die Untersuchungen ohne die Unterstützung von Frau Dr. Warmuth bei der Akquirierung von teilnehmenden Schulen, ohne die Hilfe der abgeordneten Lehrer des Berliner Netzwerks mathematisch-naturwissenschaftlich profilierter Schulen, ohne das Einverständnis der Eltern der teilnehmenden SchülerInnen und vor allem ohne den Einsatz der SchülerInnen selbst nicht möglich gewesen. Dafür danke ich sehr.

Ich bin außerdem dem Land Berlin für die finanzielle Unterstützung im Rahmen eines Elsa-Neumann-Stipendiums, der Mathematisch-Naturwissenschaftlichen Fakultät II der Humboldt-Universität zu Berlin für die Gewährung von Probandengeldern und der Berlin School of Mind and Brain der Humboldt-Universität zu Berlin für die Kostenübernahme von Konferenzteilnahmen zu Dank verpflichtet.

Abschließend möchte ich meiner Familie und meinen Freunden danken. Eure Liebe und Zuneigung gibt mir Rückhalt. Ihr seid für mich der schönste Ausgleich, wenn die Arbeit pausiert, und jeder weiß: die Pausen sind das Wichtigste.

Eidesstattliche Erklärung

Hiermit erkläre ich an Eides statt, dass ich die vorliegende Dissertationsschrift selbstständig und ohne die Benutzung unerlaubter Hilfe oder anderer als die angegebenen Hilfsmittel angefertigt habe. Alle Inhalte, die direkt oder indirekt aus veröffentlichten und nicht veröffentlichten fremden Quellen entnommen sind, sind als solche kenntlich gemacht.

Die Arbeit wurde zuvor weder in vorliegender noch in einer ähnlichen Form an dieser oder einer anderen Universität eingereicht. Weder habe ich mich anderwärts um einen Doktorgrad beworben noch besitze ich einen solchen in dem Promotionsfach Psychologie.

Mir ist die dem angestrebten Verfahren zugrunde liegende Promotionsordnung der Mathematisch-Naturwissenschaftlichen Fakultät II der Humboldt-Universität zu Berlin vom 17.01.2005, zuletzt geändert am 13.02.2006, veröffentlicht im Amtlichen Mitteilungsblatt der HU Nr. 34/2006, bekannt.

Annika Dix

Berlin, den 28.07.2015

CONTENT

ZUSAMMENFASSUNG

In Technik-geprägten Gesellschaften sind mathematische Fähigkeiten von hoher Bedeutung. Wir können Mathematikleistungen über fluide Intelligenz (FI), die Fähigkeit zu schlussfolgerndem Denken, vorhersagen. Der Einfluss von FI auf kognitive Prozesse und neuronale Mechanismen, die mathematischen Fähigkeiten in verschiedenen Teildisziplinen zugrunde liegen, ist jedoch wenig verstanden. Die vorliegende Arbeit spezifiziert FI-bezogene Unterschiede in diesen kognitiven und neuronalen Mechanismen beim Bearbeiten geometrischer Analogieaufgaben und Lösen arithmetischer und algebraischer Terme. Mithilfe eines multimethodalen Ansatzes haben wir das Zusammenspiel zwischen FI, Leistung und Faktoren wie Aufgabenkomplexität, Lernen und Strategiewahl, die kognitive Prozesse und Anforderungen beim Problemlösen beeinflussen, näher beleuchtet. Leistungsunterschiede haben wir durch Messung von Reaktionszeiten und Fehlerraten erfasst. Augenbewegungen wurden zur Strategieidentifikation erhoben. Als Indikator kortikaler Aktivität diente die ereigniskorrelierte (De-)Synchronisation (ERD/ERS) im Alpha-Band des EEG. Um kognitive Prozesse zu unterscheiden, haben wir die ERD/ERS im Theta-Band und den Unterbändern des Alpha-Bandes einbezogen. Beim Lösen unvertrauter geometrischer Analogieaufgaben zeichnete sich hohe FI durch verstärkte Verarbeitung visuell-räumlicher Informationen zum Repräsentieren von Merkmalszusammenhängen aus. Eine entsprechend erhöhte kortikale Aktivität legt nahe, dass neuronale Effizienz, als Basis hoher FI, Unterschiede im mathematischen Denken nicht erklärt. Schüler mit hoher FI passten ihre Strategiewahl den Anforderungen flexibler an, was disziplinübergreifend einer Leistungsoptimierung dienlich ist. Erstmals konnten wir aufgrund einer trialweisen Identifikation von Strategien FI-bezogene Unterschiede in der neuronalen Effizienz der Strategieausführung feststellen. Solche liegen möglicherweise Unterschieden in der Strategiewahl zugrunde. Beim Lösen vertrauter arithmetischer und algebraischer Terme zeigten sich bei Schülern mit hoher im Vergleich zu Schülern mit durchschnittlicher FI geringere Anforderungen zur Aktualisierung numerischer Repräsentationen im Arbeitsgedächtnis (AG), wohl bedingt durch die Nutzung von Routinen (z.B. Faktenabruf) anstelle mehrschrittiger Prozeduren. Ihre Leistung war deshalb in komplexen Aufgaben mit starker AG-Beanspruchung besser. Weitere Analysen lassen vermuten, dass Schüler mit hoher FI Zusammenhänge in der Aufgabenstruktur besser erkennen, um dann geeignete Routinen abrufen und auf die Struktur übertragen zu können. Die Fähigkeit hoch komplexe Zusammenhangsrepräsentationen zu bilden könnte demnach ein Schlüsselaspekt zur Erklärung FI-abhängiger Unterschiede in mathematischen Fähigkeiten sein. Die Erleichterung des Erkennens von Zusammenhängen – z.B. durch Manipulation der Aufgabendarstellung – könnte eine Möglichkeit zur Leistungsverbesserung und Unterschiedsreduktion in mathematischen Fähigkeiten darstellen.

SUMMARY

Mathematical abilities play a crucial role in our technological society. Fluid intelligence (FI), strongly related to reasoning abilities, is one of the best predictors of mathematical performance. However, the impact of FI on cognitive processes and neural mechanisms that might underlie differences in mathematical abilities across different subdivisions is not well understood. Thus, the present work sought to specify FI-related differences in the cognitive processes and neural mechanisms while students solve different mathematical problems, that is, first, unfamiliar geometric analogy tasks and second, familiar arithmetic and algebraic problems. We chose a multi-methodological approach to shed light on the interplay between FI, its associated performance, and other factors such as task complexity, level of learning, and strategy selection that influence cognitive processes and related task demands in problem-solving. We measured response times and error rates to evaluate performance differences. Eye movements were recorded to identify solution strategies. The event-related (de-)synchronization (ERD/ERS) in the broad alpha band served as indicator of general cortical activity. Further, we considered the ERD/ERS in the theta band and the three alpha sub-bands to distinguish between associated cognitive processes. For unfamiliar geometric analogy tasks, we identified a higher ability for students with high compared to average FI to build relational representations based on a more intense processing of spatial information. The associated greater cortical activity shows that neural efficiency underlying high FI is not an appropriate concept to explain FI-related differences in mathematical cognition. Strategy analyses revealed a more adaptive strategy choice in response to increasing task demands in students with high compared to average FI, which helps to optimize task performance across different mathematic subdivisions. Further, we conducted the first study identifying strategies and related cortical activity trial-wise and thereby identified FI-related differences in the neural efficiency of strategy execution. Such differences might constitute a source for FI-related differences in the strategy selection. For solving familiar arithmetic and algebraic problems, high compared to average FI was associated with lower demands on the updating of numbers probably stemming from the use of well-known routines (i.e., fact retrieval) instead of multi-step procedures. This was associated with a better performance in complex tasks relying more strongly on WM recourses. Additional analyses suggest that students with high FI had an advantage to identify the relational structure of the problems, which allows mapping and retrieving routines that match this structure. Thus, the ability to build representations of high relational complexity might be one key aspect explaining FI-related difference in mathematical abilities. We suppose that facilitating the identification of relations – for instance, by manipulating the task presentation – may improve performance and reduce FI-related differences in mathematical abilities.

List of Original Research Articles

Study I

Dix, Annika, Wartenburger, Isabell, & van der Meer, Elke. *The Role of Fluid Intelligence and Learning in Analogical Reasoning: How to Become Neurally Efficient?* Manuscript submitted for publication.

Study II

Dix, Annika, Wartenburger, Isabell, & van der Meer, Elke. *The Power of Thinking Strategically: Strategy Use Explains Differences in Neural Efficiency during Analogical Reasoning*. Manuscript submitted for publication.

Study III

Dix, Annika, Wartenburger, Isabell, & van der Meer, Elke. *How Fluid Intelligence Affects Arithmetic and Algebraic Problem-Solving: Using the Theta and Alpha ERD/ERS to Separate Cognitive Processes.* Manuscript submitted for publication.

List of Figures

List of Abbreviations

ANOVA	Analysis of variance
BOMAT	Bochumer Matrizentest
CAF	Channel alpha frequency
EEG	Electroencephalography
ERD	Event-related desynchronization
ERS	Event-related synchronization
FI	Fluid intelligence
IAF	Individual alpha frequency
ICA	Independent component analysis
PFC	Prefrontal cortex
P-FIT	Parieto-frontal integration theory
RAPM	Raven’s advanced progressive matrices
ROI	Region of interest
RT	Response time
TTL	Transistor-transistor logic
WM	Working memory

List of Symbols

%ERD/ERS	Percentage power change
α	Type I error in statistical test theory
β	1. Standardized regression coefficient
	2. Type II error in statistical test theory
A	ERD/ERS computation: test interval
M	Mean
N	Sample size
p	1. Percentile of the set of reactivity indices ρ to define the threshold of the channel selection for IAF computation
	2. p-value
R	Reference interval
$R^2_{adjusted}$	Adjusted fit index R^2 of a regression model
r	1. Fraction of the percentile of the set of reactivity indices ρ to define the threshold of the channel selection for IAF computation
	2. Pearson product-moment correlation coefficient
$r_{FI\text{-}group}$	Partial correlation with FI-group as controlling variable
SD	Standard deviation

1 Introduction

In an article on problem-solving, Simon and Newell make the following closing remark:

> There is beauty in the intricacy of human thinking when an intelligent person is confronted with a difficult problem. But there is a deeper beauty in the basic information processes and their organization into simple schemes of heuristic search that make that intricate human thinking possible. (1971, p. 159)

Highly intelligent individuals do not only arouse admiration due to their beautiful minds, but they are also more successful than less intelligent individuals due to their ability to deal with the most complex problems. A meta-analysis by Strenze (2007) revealed remarkable correlations between general intelligence but also specific components of intelligence such as fluid intelligence (FI) and a person's educational (r = .56) and occupational (r = .43) achievement indicated by the years spent in education, the highest level of education and scores on occupational scales measuring occupational status. Here, the question arises where these superior problem-solving abilities in highly intelligent individuals come from. The present work sought to specify the basic information processes that differ as a function of intelligence and might underlie individual differences in problem-solving leading to differences in academic achievement. We were especially interested in processes underlying mathematical abilities. Mathematics have a main importance in our technological society (Ashcraft & Krause, 2007). Problem-solving constitutes the center of mathematics (Halmos, 1980) and both are highly associated with FI.

1.1 Intelligence, Problem-Solving and Mathematical Abilities

Cattell (1987) defines FI as "an expression of the level of complexity of relationships which an individual can perceive and act upon when he does not have resource to answer to such complex issues already stored in memory" (p. 115). Thus, high FI is useful in particular for solving new tasks (Carpenter, Just, & Shell, 1990). FI is strongly related to reasoning abilities, such as selecting relevant or inhibiting irrelevant information to identify complex relations (Hofstadter, 1995; Holyoak & Thagard, 1995; van der Meer, 1996), to math's marks (Liepmann, Beauducel, Brocke, & Amthauer, 2007), and to the performance on the Scholastic Aptitude Test Mathematics (Gallagher, 1989).

Mathematics is characterized by an astonishing diversity, covering countless subdivisions with more than 5,000 different classes and it takes a wide range of different levels of complexity. Previous research on mathematical abilities mainly focused on one single discipline, mostly on numeracy or mental arithmetic (Floyd, Evans, & McGrew, 2003; Kyttälä & Lehto, 2008). In the present work, we were concerned with the cognitive processes during problem-solving that vary as a function of FI across different mathematical subdivisions (i.e., geometry, arithmetic, and algebra) and levels of task complexity. This is of interest as one key aspect of FI is the flexibility that enables individuals to adapt to such a variety of demands (Carpenter et al., 1990). Thus, we aimed to better understand the source of this FI-related flexibility in mathematical cognition.

1.2 Task Demands and How Working Memory Works for High FI

The solving of mathematical problems requires working memory (WM) resources (DeStefano & LeFevre, 2004; Kyttälä & Lehto, 2008), which are associated with FI (Kyllonen & Christal, 1990). WM "refers to a brain system that provides temporary storage and manipulation of the information necessary for […] reasoning" (Baddeley, 1992, p. 556). Research on WM and mathematical cognition commonly takes Baddeley's multi-component model as a basis (DeStefano & LeFevre, 2004). Accordingly, WM comprises 1) the central executive performing control functions (e.g., allocation of attention); 2) the phonological loop storing and rehearsing speech-based information; and 3) the visuospatial sketch pad storing and rehearsing visual information (Baddeley, 1986, 1992, 1996; Baddeley & Hitch, 1974)[1]. During problem-solving the involvement of each component depends on factors such as task content (e.g., numerical vs. figural relying on the phonological loop vs. visuospatial sketch pad; DeStefano & LeFevre, 2004).

Across different mathematical subdivisions, WM demands are affected by task complexity (Ayres, 2001; Ayres & Sweller, 1990; Tronsky, 2005) since task complexity is often linked to the number of steps and required cognitive processes in problem-solving. WM demands are especially high for complex multi-step problems where much information has to be maintained and processed. Moreover, task complexity affects the individuals' selection of solution strategies (Ashcraft & Krause, 2007; Bethell-Fox, Lohman, & Snow, 1984), characterized by the involvement of different cognitive processes and as a result differing in their WM demands (Hecht, 2002; Loesche, Wiley, & Hasselhorn, 2015). Another factor influencing WM demands is learning. Learning alters the involvement of cognitive processes due to changes in strategy selection (i.e., the employment of less demanding strategies). Moreover, the involved cognitive processes may get more automated (i.e., less demanding; Carpenter et al., 1990; Schoenfeld, 1992), though, a reduction of WM demands due to automatization is controversial (e.g., Tronsky, 2005).

FI is known to influence the experienced task complexity, the strategy selection and learning during mathematical problem-solving and, thus, WM demands. WM abilities are positively related to mathematical performance (for review, see Raghubar, Barnes, & Hecht, 2010) and FI (Ackerman, Beier, & Boyle, 2005), giving individuals with high compared to average or low FI an advantage especially in complex tasks. Moreover, high FI is associated with a more appropriate selection of solution strategies (Hoard, Geary, Byrd-Craven, & Nugent, 2008). These differ-

[1] Baddeley (2000) proposed a fourth component, the episodic buffer, which temporarily binds information from the other components. Since the episodic buffer plays only a minor part in most literature on mathematical cognition (DeStefano & LeFevre, 2004), it is not considered at this point.

ences in strategy selection are accompanied by differences in the involved cognitive processes and related WM demands. In addition, higher FI results in stronger learning effects (Blair, 2006; Haier, Siegel, Tang, Abel, & Buchsbaum, 1992) and decreases in required WM resources.

In sum, task demands depend on several factors such as task complexity, learning, and used strategy. They determine the set of involved cognitive processes varying in WM demands. Moreover, the three factors stated here are all related to FI. By contrast, FI-related differences in specific cognitive processes (e.g., maintaining information) are less well-understood. Studies on how such differences are reflected in mathematical performance across different subdivisions and dependent on different factors such as task complexity, learning, and strategy are pending.

1.3 Learning Analogical Reasoning and Simplifying Complex Terms

The present work aimed at specifying cognitive processes that differ dependent on FI and are related to differences in performance on mathematical problems varying in task complexity and level of learning. We chose task material including unfamiliar geometric analogy tasks (Study I and Study II) and arithmetic and algebraic problems typically found in education (Study III).

1.3.1 Cognitive Processes in Geometric Analogical Reasoning

Analogical reasoning amounts to transferring information from a source domain to a target domain based on similarities (i.e., analogies) between the two (Hofstadter, 2001). Analogies play a crucial role in mathematical cognition since problem-solving requires the mapping of representations of known problems and operations on the relational representation of the problem at hand (for review, see Dixon, 2005). Solving analogy tasks involves sub-processes such as building representations of structures, selecting relevant features, identifying relations in the source domain, transferring them to the target domain, and finally evaluating the analogy (Gentner, 1983; Holyoak & Morrison, 2005; Kokinov & French, 2003; Mulholland, Pellegrino, & Glaser, 1980). Solving geometric analogy tasks sparsely requires domain-specific knowledge and is suitable for purely measuring analogical reasoning processes (Hosenfeld, Van den Boom, & Resing, 1997).

Individuals with high compared to average FI show a better performance (shorter response times (RTs), lower error rates) when they solve geometric analogy tasks (Preusse, van der Meer, Deshpande, Krueger, & Wartenburger, 2011; van der Meer et al., 2010). These differences in performance may be associated with a different use of strategies. Studies showed that individuals with high FI spent more time on the planning phase (i.e., processes in the source domain, e.g., identifying relations) during analogical reasoning than individuals with average FI (Bethell-Fox et al., 1984; Ullwer et al., 2009). The latter tend to rely on a trial-and-error technique, where relations are mapped on the target domain to exclude false answers (execution phase). The first strategy, **constructive matching**, is more demanding but also more effective than the second,

response elimination. Earlier studies neither specified FI-related differences in individual cognitive processes involved in the execution of a strategy (e.g., identifying relations) nor did they test how such differences in analogical reasoning find expression in other mathematical subdivisions.

1.3.2 Cognitive Processes in Arithmetic and Algebraic Problem-Solving

Solving arithmetic or algebraic problems is practiced to a large extent in school. However, arithmetic and algebraic problems can be solved in many ways. Primarily, the literature distinguishes **fact retrieval** (from long-term memory) from **procedural strategies**. Fact retrieval (e.g., times tables) is cognitively less demanding than procedural strategies (De Smedt, Grabner, & Studer, 2009) and is commonly used in well-known easy tasks (Ashcraft, 1992). In comparison, procedural strategies (e.g., decomposition) are often found in more complex tasks with larger problem size (e.g., 14×17). Besides the retrieval of rules (Tolar, Lederberg, & Fletcher, 2009) and facts (e.g., after decomposition), procedures require magnitude processing (i.e., representing, assessing, and manipulating magnitudes; see also De Smedt, Noël, Gilmore, & Ansari, 2013), memory storage and updating (e.g., maintaining interim results; Passolunghi & Pazzaglia, 2004), and executive processes (e.g., planning and sequencing calculation steps; DeStefano & LeFevre, 2004; inhibiting pre-potent arithmetical responses in algebra; McNeil & Alibali, 2005).

Individuals with high compared to average FI show better performance when they solve arithmetic and algebraic problems (Dix & van der Meer, 2015). Moreover, Hoard et al. (2008) found that children with high FI use more mature strategies (e.g., decomposition instead of finger counting) and, in response to more complex problems, they shift from memory-based (i.e., high WM demands) to counting strategies, whereas children with average FI perseverate using fact retrieval without success. Thus, in accordance with the adaptive strategy choice model (Siegler & Shipley, 1995), children with high FI adaptively select backup strategies (e.g., slower counting strategies) when these help to produce correct answers during highly demanding tasks (Lemaire & Siegler, 1995). Earlier studies did not specify FI-related differences in individual cognitive processes involved in arithmetic and algebraic problem-solving (e.g. memory storage and updating) and how they are related to differences in mathematical performance.

1.4 Neural Mechanisms Underlying Mathematical Problem-Solving and High FI

There is evidence that differences in cognitive processes during mathematical problem-solving are reflected in differences in specific neural activation patterns. For instance, neural activity depends on problem size (e.g., Van Beek, Ghesquier, De Smedt, & Lagae, 2014), learning (e.g., Ischebeck et al., 2006), and the employed strategies (e.g., Grabner & De Smedt, 2011). Thus, we deemed the analysis of neural activity during problem-solving as being useful to specify FI-related differences in cognitive processes underlying mathematical abilities.

1.4.1 The Parieto-Frontal Network

Mathematical cognition involves a bilateral parieto-frontal network (Emerson & Cantlon, 2012; Preusse et al., 2011; Stocco & Anderson, 2008; Zago et al., 2001). When solving geometric analogy tasks, general frontal and parietal activity is associated with spatial encoding, WM processes like inhibiting irrelevant information (for review, see Constantinidis & Wang, 2004; Klingberg, 2006; Linden, 2007), and a categorical mapping between the source and target domain (A. E. Green, Fugelsang, & Dunbar, 2006; Wartenburger, Heekeren, Preusse, Kramer, & van der Meer, 2009). For these cognitive processes, studies do not report region-specific associations with neural activity during geometric analogical reasoning. However, the prefrontal cortex (PFC) has been associated with the integration of lower-order relations into an abstract relation (Christoff et al., 2001; A. E. Green, Fugelsang, Kraemer, Shamosh, & Dunbar, 2006; Kroger et al., 2002; Wharton et al., 2000), which is also relevant for solving typical tests measuring FI such as Raven's advanced progressive matrices (RAPM; Raven, 1958).

When solving arithmetic and algebraic problems, memory storage/updating, executive processes and retrieval from long-term memory are linked to more frontal regions and the PFC (Braver et al., 2001; Stocco & Anderson, 2008). Number processing is associated with parietal activity. Dehaene, Piazza, Pinel, and Cohen (2003) distinguished three parietal circuits that correspond to the representational systems of Dehaene's triple-code model (1992): (1) the bilateral horizontal segment of the intraparietal sulcus engages in nonverbal semantic representations of number size or distance relations (quantity system); (2) the left angular gyrus engages in verbal representations of numerals (verbal system); and (3) the bilateral posterior superior parietal lobe engages in the encoding of numbers as Arabic strings (visual system). Studies report differences in activity of these regions dependent on mathematical abilities (greater activity in the left angular gyrus for high compared do average abilities; Grabner et al., 2007), WM and FI (association with PFC functioning; for review, see Kane & Engle, 2002; greater activity in the parietal sulcus for high compared to average FI; K. H. Lee et al., 2006). One assumption used to explain FI-related differences in neural activity is that neural efficiency underlies high FI.

1.4.2 Neural Efficiency and the Event-Related (De-)Synchronization as Neural Correlates of Cognitive Processes in Mathematical Problem-Solving

According to the parieto-frontal integration theory (P-FIT; R. E. Jung & Haier, 2007), high FI is accompanied by a different activation of the parieto-frontal network. The neural efficiency hypothesis by Haier et al. (1988) states that "intelligence is not a function of how hard the brain works but rather how efficiently it works" (Haier, Siegel, Tang, et al., 1992, pp. 415-416). Thus, individuals with high compared to average FI were assumed to show a reduced brain activity, which is, however, moderated by factors such as task difficulty, familiarity, learning, and brain region (for review, see Neubauer & Fink, 2009a). Briefly, neural efficiency is particularly present in easy, familiar tasks and mostly concerns frontal brain regions.

The event-related (de-)synchronization (ERD/ERS) in the alpha band of the electroencephalogram (EEG; for description, see 3.2), indicating general cortical activity (Klimesch, 1999), was used in several studies to determine the impact of FI on the recruitment of the parieto-frontal network and its neural efficiency (e.g., Neubauer & Fink, 2009b). In addition, the ERD/ERS in the alpha sub-bands and in the theta band can be used to distinguish between different cognitive processes. The upper alpha ERD (ca. 11-13 Hz) is topographically restricted to task-relevant brain areas and correlates with task-specific processes. For instance, Klimesch, Schimke, and Schwaiger (1994) reported a left-hemispheric upper alpha ERD during a semantic analogy task, which can be associated with semantic memory processes (Martin & Chao, 2001). By contrast, the widespread lower alpha ERD measures attentional processes. More precisely, the lower-1 alpha ERD (ca. 7-9 Hz) refers to alertness/arousal and the lower-2 alpha ERD (ca. 9-11 Hz) to expectancy (Klimesch, Doppelmayr, Russegger, Pachinger, & Schwaiger, 1998). However, to our knowledge, there are no studies specifying the relation between these ERD/ERS measures and cognitive processes during geometric analogy tasks and how processes are affected by FI.

For arithmetic problem-solving, the following associations between different ERD/ERS measures and cognitive processes are known: (1) a left-hemispheric fronto-central and parieto-occipital theta ERS reflects fact retrieval; (2) a bilateral parieto-occipital upper alpha ERD or a widespread lower alpha ERD reflect procedural strategies involving magnitude processing, memory storage/updating, and executive processes (De Smedt et al., 2009; Grabner & De Smedt, 2011); (3) a lower-1 alpha ERS reflects memory storage/updating (Jensen & Tesche, 2002); and (4) a frontal theta ERS reflects executive processes (Klimesch, Sauseng, & Hanslmayr, 2007). However, to our knowledge, there are no studies on arithmetic and algebraic problem-solving that evaluate the impact of FI on these ERD/ERS measures and associated cognitive processes. Evaluating differences in these ERD/ERS measures provides a promising approach to determine those associated cognitive processes that are affected by FI and might constitute a source of individual differences in mathematical abilities.

1.4.3 Differences in Neural Efficiency: The Impact of Strategy Selection and Execution

Poldrack (2015) recently pointed out that studies taking differences in brain activity (i.e., neural efficiency) as basis for explaining individual differences face a serious problem as they rely on the assumption that individuals perform the same computations for problem-solving without testing it. Thus, they cannot distinguish between differences in neural activity stemming from the use of different strategies (and corresponding cognitive processes) and from differences in the execution of the same strategy/cognitive processes. For instance, learning is known to induce changes in strategy (Lemaire & Siegler, 1995). Since different strategies are associated with different brain activation patterns (Glabus, 2003; Grabner & De Smedt, 2011; K. Lee et al., 2010), learning-related changes in brain activity are often assumed to reflect changes in the strategy selection (Bernstein, Beig, Siegenthaler, & Grady, 2002; Haier, Siegel, MacLachlan, et al., 1992; Kelly & Garavan, 2005). However, according to Jonides (2004), learning-related changes in brain activity might also reflect an improvement to execute the initial strategy.

For geometric analogy tasks, studies indicate that students with average and high FI rely on the use of different strategies. Bethell-Fox et al. (1984) showed that high FI is associated with a more frequent use of the more demanding strategy constructive matching to solve geometric analogy tasks. Moreover, the lower the FI score the more often individuals change the strategy in difficult tasks from constructive matching to response elimination. More recent studies do not report such strategy changes (Ullwer et al., 2009; Vigneau, Caissie, & Bors, 2006) and, in other mathematical subdivisions such as arithmetic, strategy changes for problem-solving even occur more often for individuals with high compared to average FI (Hoard et al., 2008). These changes were interpreted as a more adaptive strategy choice for high FI (cf. adaptive strategy choice model; Siegler & Shipley, 1995). Though the exact differences in strategy use dependent on FI need to be further defined, especially differences in the adaptive use according to task demands, all these earlier studies suggest that FI affects strategy selection. Thus, differences in the cortical activity might result from the use of different strategies and/or from differences in the execution of the same strategy. This emphasizes the need to test individual differences in strategy selection and to control for such differences when evaluating FI-related differences in neural efficiency during problem-solving.

In conclusion, research on FI-related differences in individual cognitive processes during mathematical problem-solving and their generalizability across different subdivisions remains pending. Moreover, studies on the neural mechanisms that are associated with these cognitive processes and might underlie mathematical abilities do not consider FI-related differences in strategy use and, thus, do not allow making clear conclusions about FI-related differences in the execution of strategies and related cognitive processes.

2 Research Questions and Hypotheses

The present work aimed at specifying cognitive processes and underlying neural mechanisms that differ as a function of FI and might be related to differences in performance when individuals solve mathematical problems from different subdivisions (geometry, arithmetic, and algebra) that vary in task complexity and familiarity. Thus, this work sought to contribute to a better understanding of the flexibility that enables individuals with high FI to adapt to the variety of different demands in mathematical problem-solving.

Analogical reasoning is a core process in problem-solving (Hofstadter, 2001) and plays an important role in mathematical cognition (Dixon, 2005). Studies on geometric analogical reasoning revealed effects of FI (Preusse et al., 2011) and short-term learning (Wartenburger et al., 2009) on performance and neural activity. In these studies subjects were highly familiar with the task and learning effects were not assessed comparing different levels of FI. However, high FI particularly enables to deal with new tasks (Carpenter et al., 1990) and Haier, Siegel, Tang, et al. (1992) showed that learning-related increases in neural efficiency are positively correlated with FI. Especially during early phases of learning, where WM resources are of particular importance, FI should influence learning-induced changes (Ackerman, 1987, 1988). Thus, in **Study I**, we addressed the following main questions: Are there FI-related differences in performance and neural efficiency and their change during the early phase of (short-term) learning when individuals with average and high FI solve **unfamiliar** geometric analogy tasks? With which cognitive processes and underlying neural mechanisms are these differences associated?

- *Hypothesis I-I:* Individuals with high FI outperform individuals with average FI (shorter RTs, fewer errors). Performance improves across the experiment (shorter RTs, fewer errors in the second compared to the first half of the experiment).
- *Hypothesis I-II:* FI affects neural efficiency (Preusse et al., 2011) defined as amount of cortical activity. Individuals with high compared to average FI show for the broad alpha band a greater parieto-occipital ERD, indicating greater cortical activity (Klimesch, 1999), and a smaller frontal ERD, indicating smaller cortical activity.
- *Hypothesis I-III:* The ERD in the broad alpha band decreases across the experiment resulting in a smaller alpha ERD in the second half of the experiment compared to the first, especially for individuals with high compared to average FI.
- Addendum: We differentiated between the two lower and the upper alpha band (for specification, see 1.4.2) to identify cognitive processes (building representations, selecting features, identifying relations, mapping relations, evaluating the analogy; see 1.3.1) that are associated with differences in neural efficiency dependent on FI and short-term learning.

Recently, Poldrack (2015) emphasized the need to consider differences in strategy use when evaluating FI-related differences in neural efficiency. Bethell-Fox et al. (1984) showed that the higher FI was, the more often individuals selected constructive matching to solve geometric analogy tasks. Further, the lower the FI score was, the more often individuals changed the strategy from constructive matching to response elimination for difficult tasks. The authors analyzed eye movements to identify different strategies. However, findings on the occurrence of strategy changes are inconsistent (Ullwer et al., 2009; Vigneau et al., 2006). In other mathematical subdivisions, high, but not average FI, is associated with strategy changes with increasing task demands allowing for better performance (Hoard et al., 2008). According to Poldrack (2015), differences in neural activity might stem from the use of different strategies or from differences in the execution of the same strategies. To evaluate FI-related differences in the execution of strategies, we need to control for differences in strategy use (see also Grabner & De Smedt, 2011). Thus, in **Study II**, we addressed the following main questions: Are FI-related differences in strategy use and related performance apparent when individuals with average and high FI solve an unfamiliar geometric analogy task? When controlling for differences in strategy selection, are there FI-related differences in neural efficiency indicating differences in strategy execution?

- *Hypothesis II-I:* Eye movements and associated strategies differ dependent on FI. An increase in error rates accompanies strategy changes dependent on task difficulty in individuals with average FI (Bethell-Fox et al., 1984) but not with high FI (Hoard et al., 2008).
- *Hypothesis II-II:* When controlling for strategy, a greater parieto-occipital ERD and a smaller frontal ERD in the broad alpha band (see *Hypothesis I-II*) indicate differences in neural efficiency of strategy execution.

In the long-run, high FI might facilitate acquiring task-relevant knowledge and procedures (Blair, 2006). Individuals with high FI solve familiar arithmetic and algebraic tasks faster and more accurately than individuals with average FI (Dix & van der Meer, 2015). Hoard et al. (2008) found that FI affects strategy selection in arithmetic problem-solving. The use of different strategies is accompanied by a different involvement of task-relevant cognitive processes such as fact retrieval, magnitude processing, memory storage/updating, and executive processes. However, studies that determine the individual relation between these cognitive processes and FI in order to identify sources of individual differences in performance are pending. Thus, in **Study III**, we addressed the following main questions: Which cognitive processes and underlying neural mechanisms are affected by FI when individuals solve **familiar** arithmetic and algebraic problems with varying complexity? How are these differences related to performance?

- *Hypothesis III-I:* Individuals with high FI outperform individuals with average FI (shorter RTs, fewer errors).
- *Hypothesis III-II:* Individuals with high FI use fact retrieval more often (Hoard et al., 2008) than individuals with average FI, show a superior processing of magnitudes (Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & Van de Rijt, 2009), and an advantage in memory storage/updating, and executive processes (Friedman et al., 2008). These differences in cognitive processes are reflected in the ERD/ERS of the theta band and the three alpha sub-bands with the above mentioned associations (for specification, see 1.4.2).

3 Methods

3.1 Participants and Testing

Participants were students recruited within a project of the Berlin Senate Department and the Department of Mathematics at Humboldt-Universität zu Berlin supporting mathematically gifted students from Berlin schools specialized in mathematics and natural sciences (http://www-didaktik.mathematik.hu-berlin.de/netzwerk.html). Participating students ($N = 110$; 44 female; age: $M = 15.8$ years; 10th Grade) and their parents gave informed written consent prior to the investigation. Each study was approved by the ethics committee of the Humboldt-Universität zu Berlin and followed American Psychological Association standards and the Declaration of Helsinki (World Medical, 2013).

First, students underwent psychometric testing (group testing) in the school facilities. Students gave their age, gender, and grades in the ninth Grade. They stated the highest professional qualification of their parents as an indicator of the students' socioeconomic background and answered questions about parental support on a five-point Likert scale. Afterwards, each student's FI was assessed with the Bochumer Matrizentest – advanced (BOMAT; Hossiep, Hasella, & Turck, 2001). According to their BOMAT scores, we assigned students with raw scores below 17 – which corresponds to a Sten score of 7 ($M = 5.5$, $SD = 2$) in a highly selective norm sample consisting of above average intelligent individuals – to the group of students with average FI and students with raw scores of 17 or higher to the group of students with high FI. A subsample of both groups took part in the two follow-up EEG experiments, controlling for socioeconomic background and the students' ratings of parental support. All students had normal or corrected-to-normal vision, no neurological or psychiatric diseases and were not taking any medication. Students were paid for their participation (€ 8.00 per hour).

Experiment I (duration: 3.5 hours) took place five months after psychometric testing, with 79 participants being included in the final analyses for Study I (28 female[2]; age: $M = 15.78$ years, $SD = 0.54$). A subsample of 49 students, for which eye movements were recorded during testing, were included in the final analyses for Study II (18 female; age: $M = 15.82$ years, $SD = 0.50$). Experiment II (duration: 2.5 hours) took place 12 months after psychometric testing, with 60 participants being included in the final analyses for Study III (23 female; age: $M = 15.75$ years, $SD = 0.55$). Both experiments were individual testing sessions in an EEG laboratory, where participants sat in front of a monitor at a distance of 50 cm (size: 18″; resolution: 1,280 × 1024).

3.2 EEG and the ERD/ERS

The EEG is a non-invasive measurement of voltage fluctuations along the scalp reflecting electrical activity within the neurons of the brain (Niedermeyer & Lopes da Silva, 2004). The measured brain activity in the EEG consists of different brain rhythms or oscillations, that is, regularly

[2] Gender has been shown to influence the relationship between intelligence and neural efficiency (e.g., Neubauer, Grabner, Fink, & Neuper, 2005). To control for an unequal gender distribution in the two groups, we also ran all EEG analyses incorporating gender as a covariate. Results are comparable to those without the covariate gender. For reasons of clarity and comprehensibility we only report results from the simpler analyses without covariate.

recurring waves which can be classified according to their frequency. Cognitive processes are accompanied by changes of these oscillations, for instance, changes in phase or amplitude. Changes within a certain frequency range or band are assumed to reflect the activation of functional networks of the brain (Klimesch, Schack, & Sauseng, 2005). The ERD/ERS method quantifies task-related changes in power (i.e., amplitude) by comparing the power during a task (test interval, A) with the power during a preceding reference interval (R) at rest (Pfurtscheller & Aranibar, 1977; Pfurtscheller & Lopes da Silva, 1999). We describe the definition of intervals for tasks used in the present work in 4.2.1 and 6.2.1. Calculating the percentage change in the power with %ERD/ERS = (R-A)/R, gives positive values when power decreases (desynchronizing neurons; ERD), and negative values when power increases (synchronizing neurons; ERS). We used the ERD/ERS in the alpha band (ca. 8-13 Hz) as indicator of general cortical activity (Klimesch, 1999) and, in addition, the ERD/ERS in the three alpha sub-bands and the theta band (ca. 4-7 Hz) to distinguish between associated cognitive processes (for description, see 1.4.2).

Individuals' alpha frequency is affected by several external and internal factors such as task demands (Klimesch, Schimke, & Pfurtscheller, 1993) and FI (Anokhin & Vogel, 1996). The use of fixed frequency bands for ERD/ERS computation can lead to distorted estimations of changes in band power (e.g., for individuals with exceptional slow or fast alpha rhythms). Thus, we determined each participant's individual alpha frequency (IAF) and defined the theta and alpha bands individually according to the frequency ranges described in Klimesch (1999). For that, we recorded participants' resting state EEG first with closed eyes and, second, with opened eyes at the beginning of Experiment I. With open eyes alpha power desynchronizes (Berger, 1930) and the responsiveness region of the alpha rhythm can be measured. We used a channel reactivity based (CRB) algorithm proposed by Goljahani et al. (2012), which first identifies the alpha responsiveness region and then determines the spectral gravity center over this region for each channel (channel alpha frequency, CAF). We defined the IAF as mean of all CAFs.

3.3 Eye Movements

Several studies analyzed eye movements to evaluate strategic differences in solving geometric analogy tasks (e.g., Vigneau et al., 2006). Fixations indicate the focus of visual attention (Just & Carpenter, 1976). The frequency (i.e., number) and duration of fixations on task-relevant areas are positively correlated with problem-solving abilities (H. J. Green, Lemaire, & Dufau, 2007). The scan path is an indicator of the information search process during problem-solving (Sprague & Ballard, 2003). For geometric analogy tasks, Bethell-Fox et al. (1984) used these different eye movement parameters to distinguish between strategies. Constructive matching was characterized by a smaller frequency but longer duration of fixations compared to response elimination.

The total scan path length and fixation duration on the source domain during planning was longer for constructive matching than for response elimination. In accordance with Landgraf et al. (2011), using similar stimuli, we counted midline crossings (i.e., the number of crossings between source and target domain) to identify strategies. One midline crossing indicates the employment of constructive matching, more than two midline crossings the employment of response elimination.

We followed the approach of Dimigen, Sommer, Hohlfeld, Jacobs, and Kliegl (2011) for the co-registration of data. The EEG- and eye movement data were thoroughly synchronized using shared transistor-transistor logic (TTL) trigger. We avoided contact pressure artifacts in the EEG from the forehead rest of the eye-tracker by fixing a foam-cushion on participants' forehead and muscle artifacts from neck muscles by carefully adapting participants' seating position. To account for electromagnetic artifacts from the eye-tracker (50 Hz) we applied a notch filter to the EEG data. After performing an Independent Component Analysis (ICA), we used a component-based artifact rejection (T.-P. Jung et al., 2000) to correct for corneoretinal and myogenic eye movement artifacts in the EEG. The luminance of stimuli was controlled for and the contrast within and between stimuli and the light-grey background was minimized to reduce saccade-related differences in the amplitude of the visual-driven lambda response (Kazai & Yagi, 2005), potentially affecting the EEG power.

4 Summary of Study I "The Role of Fluid Intelligence and Learning in Analogical Reasoning: How to Become Neurally Efficient?"

4.1 Theoretical Background

Study I evaluates the impact of FI on adaptive changes in neural efficiency due to short-term learning while solving an unfamiliar geometric analogy task. Analogical reasoning is a core process in problem-solving (Hofstadter, 2001). In a **familiar** geometric analogy task, Preusse et al. (2011) showed that high compared to average FI was associated with better performance and greater activity in parieto-occipital brain regions but smaller activity in frontal brain regions. However, learning also affects neural efficiency (for review, see Neubauer & Fink, 2009a) and the impact of learning (i.e., increasing neural efficiency; Wartenburger et al., 2009) might vary dependent on FI, with greater activity decreases in individuals with high FI than in those with average FI during learning (Haier, Siegel, Tang, et al., 1992; Neubauer, Grabner, Freudenthaler, Beckmann, & Guthke, 2004).

The purpose of this study was to evaluate the impact of FI and short-term learning on neural efficiency when solving **unfamiliar** geometric analogy tasks. By analyzing the alpha ERD, indicating cortical activity (Klimesch, 1999), while differentiating between the alpha sub-bands, we aimed to specify cognitive processes (e.g., identifying relations) and neural mechanisms underlying FI-related differences in analogical reasoning abilities.

4.2 Methods

Participants with average ($N = 37$) and high ($N = 42$) FI solved unfamiliar geometric analogy tasks. Each trial started with a fixation cross displayed in the center of the screen for 3,000 ms (reference interval: from 1,500 ms until 2,500 ms after stimulus presentation). Afterwards, the analogy task appeared consisting of two pairs of patterns, a source pair and a target pair (see Figure 1). Within each pair, patterns were presented in one out of five alignments: mirrored at the vertical, the horizontal, the major (falling) or minor (rising) diagonal, or no mirroring at all (identical condition). Participants were instructed to decide as quickly and as accurately as possible whether both pairs showed the same type of relation (analogy, 50%) or different types of relation (distractor, 50%). The test interval for the %ERD/ERS computation was defined as the time between task presentation and the pressing of the response button. A variable inter-trial interval (250-750 ms) with a centered fixation cross completed a trial.

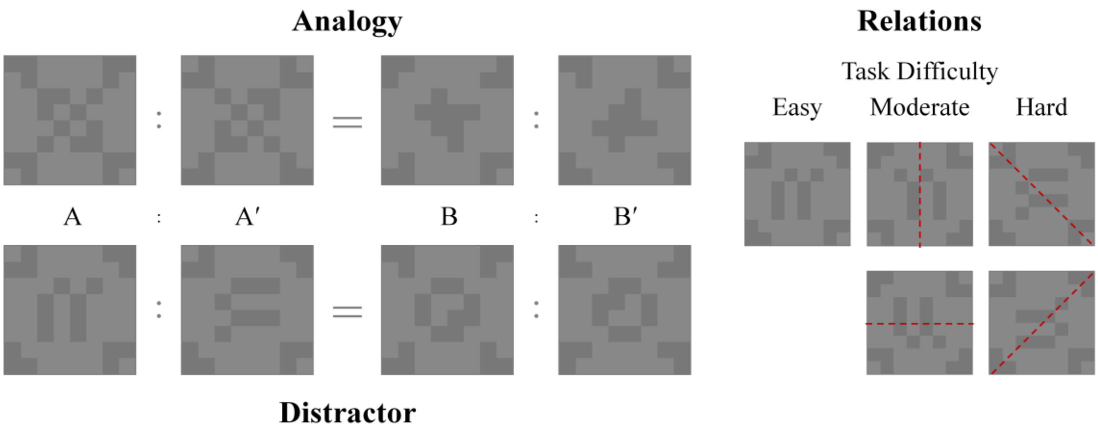

Figure 1. Example of an analogy item: horizontal relation in source (A : A') and target (B : B') pair (top left); a distractor item: major diagonal relation in the source and minor diagonal relation in the target pair (bottom left); and the five possible alignments (right): identical (easy), mirrored at the vertical and the horizontal (moderate), and mirrored at the major (falling) and minor (rising) diagonal (hard).

RTs, error rates, and the EEG were recorded as dependent variables. Technical and myogenic artifacts were removed from EEG data. Error trials, outliers (i.e., trials with exceptional short or long RTs) and distractor trials were not considered for the analyses. We performed mixed-design analyses of variance (ANOVAs) on behavioral data and the %ERD/ERS for the broad alpha band and the alpha sub-bands while distinguishing between easy (identical relation), moderate (vertical and horizontal relations), and hard (both diagonal relations) tasks and between trials from the first half of the experiment and trials from the second half. For EEG data, we compared effects for the two hemispheres and three regions of interest (ROI; frontal: Fp1/2, AF3/4, F3/4

F7/8, FC1/2; central: FC5/6, FT7/8, C3/4, CP5/6, TP7/8; parieto-occipital: P3/4, P7/8, PO3/4, PO7/8, PO9/10).

4.3 Results

First, high FI was associated with a faster and more accurate performance than average FI, which was especially pronounced in difficult tasks. In the broad alpha band, participants with high compared to average FI showed greater right-hemispheric cortical activity in hard tasks. In the lower-2 alpha band, participants with high FI exhibited greater cortical activity over left-central brain regions than participants with average FI.

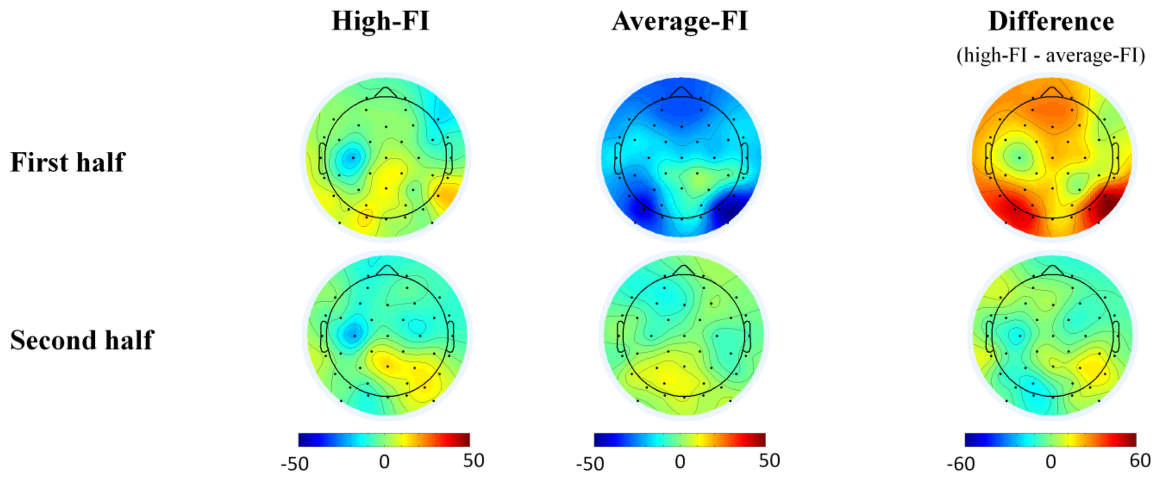

Figure 2. Topographic distribution of the alpha %ERS/ERD during hard tasks for students with high (high-FI; left) and average fluid intelligence (average-FI; middle) and the difference between high FI and average FI (right) in the first (top) and second half (bottom) of the experiment.

Second, learning led to a faster and more accurate performance in the second half of the experiment compared to the first half, especially in difficult tasks. In the broad alpha band, learning in participants with average FI was associated with increased right-hemispheric cortical activity for hard tasks (see Figure 2). For participants with high FI, learning was associated with decreased right-hemispheric cortical activity for moderate tasks. In the lower-1 alpha band, we found a greater left- than right-hemispheric cortical activity for both groups in the first half of the experiment, but not in the second half.

4.4 Discussion

This study on the solving of unfamiliar geometric analogy tasks shed light on cognitive processes that are influenced by FI and short-term learning and associated with differences in performance and neural efficiency. The results support *Hypothesis I-1* on FI and learning-related differences in performance, with a better performance for students with high compared to average FI (see also van der Meer et al., 2010) and a learning-related improvement in performance from the first to the second half of the experiment (see also Carpenter et al., 1990).

Hypothesis I-II about FI-related differences in neural efficiency is only partly supported. Right-hemispheric cortical activity in the alpha band was greater for students with high compared to average FI over parietal regions (see also Preusse et al., 2011) but also frontal regions and only for hard tasks. By analyzing the alpha sub-bands, we could attribute the FI-related differences in cortical activity to cognitive processes reflected in the lower-2 alpha band suggesting FI-related differences in attentional processes (see also Klimesch et al., 1998).

Hypothesis I-III about learning-induced increases in neural efficiency was supported for students with high FI. For students with average FI, neural efficiency decreased from the first to the second half of the experiment. In the alpha sub-bands, learning-related differences in cortical activity were restricted to cognitive processes reflected in the lower-1 alpha band suggesting a learning-related change in expectancy formation (see also Klimesch et al., 1998). See 7.2 and 7.4.3 for further discussion.

5 Summary of Study II "The Power of Thinking Strategically: Strategy Use Explains Differences in Neural Efficiency during Analogical Reasoning"

5.1 Theoretical Background

Study II evaluates the impact of FI on strategy selection and neural efficiency of strategy execution while solving unfamiliar geometric analogy tasks. Earlier studies on FI-related differences in neural efficiency are based on the assumption that individuals perform the same computations for problem-solving, an assumption usually not tested and maybe not true (Poldrack, 2015). Actually, earlier studies on geometric analogical reasoning suggest FI-related differences in strategy selection (Bethell-Fox et al., 1984; Ullwer et al., 2009; Vigneau et al., 2006), though the exact differences in strategy use dependent on FI need to be further defined, especially differences in the adaptive use according to task demands (Hoard et al., 2008; Siegler & Shipley, 1995).

Preusse et al. (2011) found FI-related differences in the neural activity when students solve geometric analogy tasks indicating differences in neural efficiency. However, FI-related differences in the type of gestures that were observed during strategy reports after the experiment by Preusse et al. (2011) pointed to differences in the cognitive processes and corresponding strategies selected for problem-solving (Sassenberg, Foth, Wartenburger, & van der Meer, 2011). Thus, the purpose of this study was twofold: first, we aimed at evaluating the impact of FI on strategy use for solving unfamiliar geometric analogy tasks. Second, we sought to distinguish between FI-related differences in neural efficiency stemming from the use of different strategies and those due to differences in the execution of the same strategies by controlling for strategy (see also Grabner & De Smedt, 2011).

5.2 Methods

Participants with average ($N = 25$) and high ($N = 24$) FI solved geometric analogy tasks of varying difficulty (for description, see 4.2). The size of the patterns in this task corresponded to a visual angle of 9.16° × 9.16°. RTs, error rates, eye movements and the EEG were recorded as dependent variables. Blinks were excluded from the data and we only analyzed fixations towards the patterns (square area that was 0.36° smaller than the pattern; gaze position accuracy: ca. 0.25°). Data processing was analogous to Study I. However, instead of considering effects of learning, we included the effect of strategy in the analyses by distinguishing between trials, in which constructive matching versus response elimination was employed (see Figure 3; for description, see 3.3).

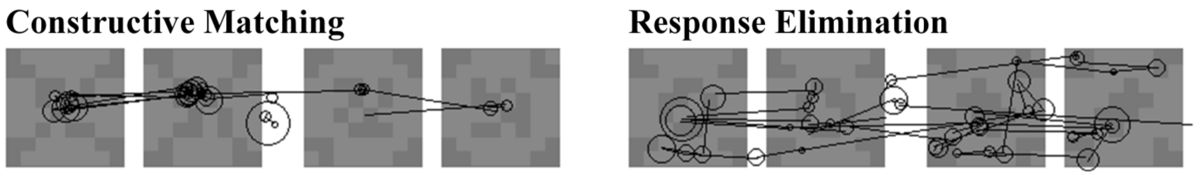

Figure 3. Illustration of a trial, in which constructive matching was employed (one midline crossing; left), and a trial, in which response elimination was employed (more than two midline crossings; right); black lines represent participants' scan path, black circles illustrate fixations (the greater the circle, the longer the fixation duration).

The validity of strategies for the two groups were tested using ANOVAs on behavioral data and eye movements (mean fixation duration, total number of fixations, total scan path length and fixation duration before the first midline crossing). Differences in strategy use for the two groups and different levels of task difficulty were tested using ANOVAs on RTs, error rates and percentage use of constructive matching. Differences in neural efficiency for the two groups were tested performing an ANOVA on %ERD/ERS in the alpha band. A similar ANOVA, additionally comparing the different levels of task difficulty, was conducted but only with trials, where constructive matching was employed (17 participants did not employ response elimination for every level of task difficulty)

5.3 Results

The strategy validation revealed longer mean fixation durations for students with high compared to average FI. Concerning strategy use, participants with high FI changed their strategy from constructive matching to response elimination in moderate tasks. Participants with average FI employed constructive matching less often only in hard compared to easy and moderate tasks. While constructive matching was the more effective strategy (faster, more accurate) than response elimination, the accuracy of constructive matching in moderate tasks was lower for participants with average compared to high FI. Error rates increased with increasing task difficulty for participants with average FI (see Figure 4a). Concerning neural efficiency, the amount of cortical activity did not differ between groups (see Figure 4b). However, opposed to participants with average FI, those with high FI activated the right hemisphere more strongly than the left.

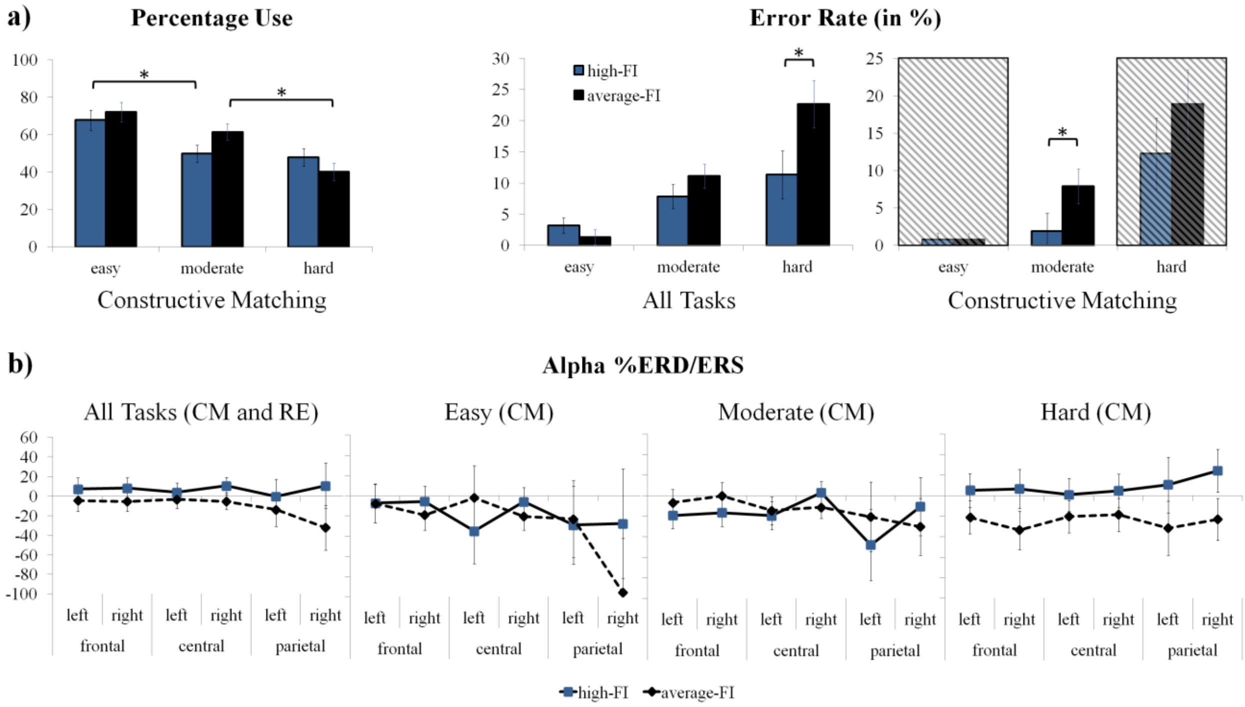

Figure 4. **a.** Effect (* = significant difference) of task difficulty (easy, moderate, hard) and group (students with high fluid intelligence, high-FI; students with average fluid intelligence, average-FI) on percentage use (± 1SE) of constructive matching (CM; left) and error rates (± 1SE) for all tasks (middle) and post-hoc for moderate tasks (we shaded easy and hard tasks as they were not included in the analysis), in which CM was employed (right); and **b.** %ERD/ERS scores (± 1SE) for each group, hemisphere (left versus right), and ROI (frontal, central, parietal = parieto-occipital) displayed (from left to right) for both strategies (CM, RE) and dependent on task difficulty (easy, moderate, hard) for trials where CM was employed.

5.4 Discussion

This study shed light on FI-related differences in strategy use, the related performance and neural efficiency while solving unfamiliar geometric analogy tasks. *Hypothesis II-I* on FI-related differences in eye movements indicating differences in strategy use was supported. Students with high FI changed their strategy from constructive matching to response elimination for moderate and hard tasks, whereas strategy changes for students with average FI only occurred for hard tasks. The lower accuracy for employing constructive matching in moderate tasks and the increasing error rates in hard tasks for students with average FI indicate less adaptive strategy choices (see Siegler & Shipley, 1995) for these students compared to their highly intelligent peers. Strategy use was qualitatively comparable for the two groups, except the greater mean fixation duration in students with high compared to average FI.

Hypothesis II-II about FI-related differences in neural efficiency when controlling for strategy use is (partly) supported. We did not find FI-related differences in the amount of cortical activity – even when considering the different levels of task difficulty for constructive matching.

However, the right-lateralization for students with high FI and the left-lateralization for students with average FI suggest that FI-related differences in cortical activity, we found in Study I, partly refer to differences in the neural efficiency of strategy execution (for further discussion, see 7.3).

6 Summary of Study III "How Fluid Intelligence Affects Arithmetic and Algebraic Problem-Solving: Using the Theta and Alpha ERD/ERS to Separate Cognitive Processes"

6.1 Theoretical Background

Study III evaluates the impact of FI on the cognitive processes and neural correlates that underlie the solving of familiar arithmetic and algebraic problems varying in task complexity. FI has been shown to affect the selection of strategies (Hoard et al., 2008) and, thus, partly determines which cognitive processes are involved in problem-solving. In arithmetic, we distinguish between fact retrieval and procedural strategies. Procedural strategies involve cognitive processes such as magnitude processing (De Smedt et al., 2013), information storage/updating and executive processes (for review, see DeStefano & LeFevre, 2004). Since WM resources are limited (Baddeley & Hitch, 1974) and positively correlated with FI (Ackerman et al., 2005) high FI might optimize performance by relying more strongly on processes with low WM demands (i.e., fact retrieval) or by providing more resources to execute processes with high WM demands.

We were interested in the impact of FI on these cognitive processes in arithmetic and algebraic problem-solving. The processes are related to different ERD/ERS measures in the alpha sub-bands and in the theta band: (1) a left-hemispheric fronto-central and parieto-occipital theta ERS reflects fact retrieval; (2) a bilateral parieto-occipital upper alpha ERD or widespread lower alpha ERD reflect procedures including magnitude processing, memory storage/updating, and executive functions (De Smedt et al., 2009; Grabner & De Smedt, 2011); (3) a lower-1 alpha ERS reflects memory storage/updating (Jensen & Tesche, 2002); and (4) a frontal theta ERS reflects executive functions (Klimesch et al., 2007). We determined the impact of FI on these ERD/ERS measures to separate the cognitive processes and to evaluate their relations to FI.

6.2 Methods

Participants with average ($N = 30$) and high ($N = 30$) FI solved familiar arithmetic and algebraic problems (see also Dix & van der Meer, 2015). Each trial started with a fixation cross displayed in the center of the screen for 3,000 ms (reference interval: from 1,500 ms until 2,500 ms after stimulus presentation). Afterwards, the problem appeared belonging to one of six types of problems varying in complexity (see Figure 5). Participants were instructed to mentally solve the problem as quickly and as accurately as possible and then to request candidate answers by pressing a button. The test interval for the %ERD/ERS computation was defined as the time between presentation of the problem and button press. After that, four candidate answers appeared arranged around an "X" in the center of the screen. Participants had to select the answer that matched their solution. The X had to be selected if none of the four candidate answers matched the participants' solution (i.e., distractors; 20%). A variable inter-trial interval (250-750 ms) with a centered fixation cross completed a trial.

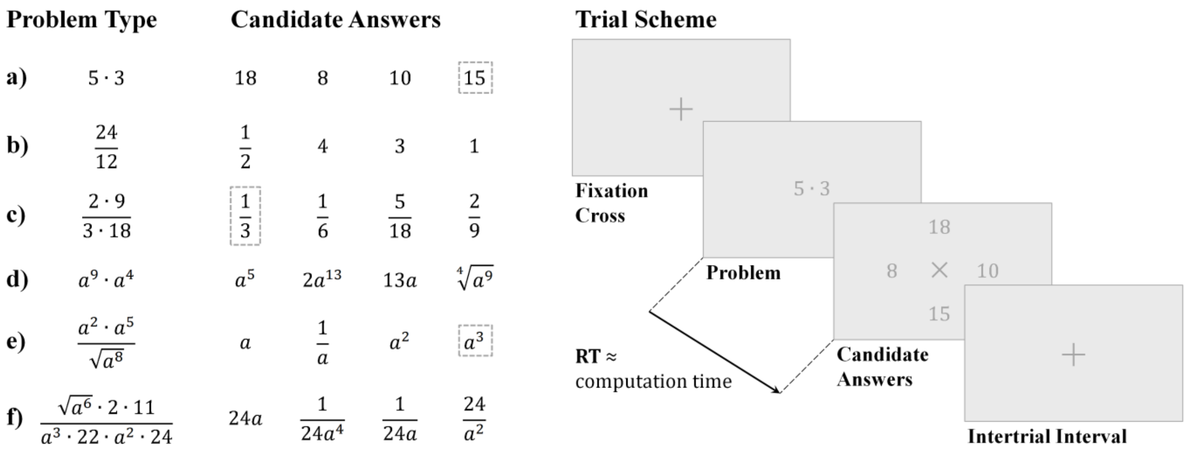

Figure 5. Examples of arithmetic and algebraic expressions (left) with corresponding candidate answers (correct answers framed; middle) and a schematic illustration of a trial from left to right starting with the presentation of a fixation cross (right); arithmetic problem types with increasing task complexity: a) multiplication of two one-digit numbers or one one-digit and one two-digit number, b) canceling down of two one-digit or two-digit numbers, c) operations with fractions (a and b combined); algebraic problem types with increasing task complexity: d) simplification of basic algebraic expressions, e) simplification of advanced algebraic expressions; combined arithmetic and algebraic problem type: f) simplification of complex algebraic expressions (c and e combined).

RTs, error rates, and the EEG were recorded as dependent variables. Technical and myogenic artifacts were removed from EEG data. Error trials, outliers and distractors were not considered for the analyses. Behavioral data and %ERD/ERS for the theta band and the alpha sub-bands were analyzed using ANOVAs distinguishing between the six problem types. For EEG data, we compared effects for the two hemispheres and two ROIs (fronto-central: FC1/2, FC5/6, C3/4; parieto-occipital: PO3/4, PO7/8, O1/2).

6.3 Results

First, participants with high compared to average FI solved the more complex problems (type e and f) faster. Second, cortical activity was greater over parieto-occipital regions than over fronto-central regions in all alpha sub-bands. In the upper alpha band, the ERD in participants with high FI was greater over the right hemisphere than over the left, but only for less complex problems. For the most complex problems (type f), the ERD was greater over the left hemisphere than over the right. The inverse activation patterns over the hemispheres were observed for participants with average FI (see Figure 6a). In the lower-1 alpha band, participants with high compared to average FI exhibited a smaller parieto-occipital ERD (see Figure 6b).

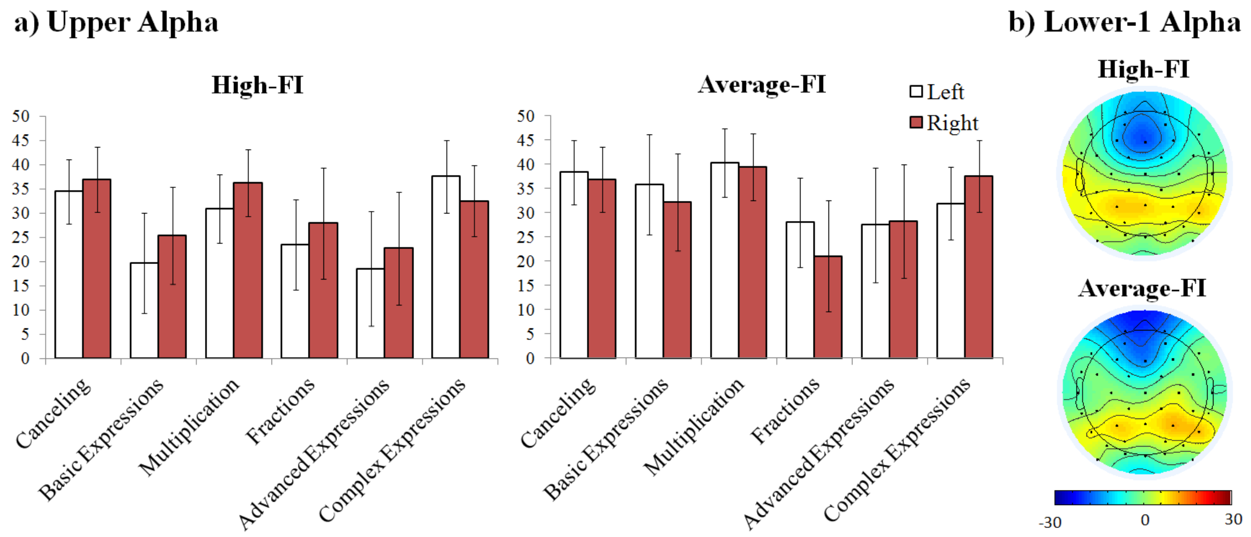

Figure 6. **a.** Upper alpha %ERD/ERS scores (± 1SE) for each group (students with high fluid intelligence, high-FI; students with average fluid intelligence, average-FI), hemisphere (left versus right), and level of task complexity (six problem types; increasing complexity from left to right); and **b.** topographic distribution of the lower-1 alpha %ERD/ERS for students with high (top) and average FI (bottom) averaged across all problem types.

6.4 Discussion

This study shed light on FI-related differences in cognitive processes during the solving of familiar arithmetic and algebraic problems. The behavioral results partly support *Hypothesis III-I* on performance differences. The high familiarity of the problems might explain that students with high compared to average FI performed better (shorter RTs) only on complex problems, which require additional memory storage/updating and executive processes.

In line with *Hypothesis III-II*, we found FI-related differences in cortical activity particularly referring to memory storage/updating. The greater left-hemispheric upper alpha ERD in students with high FI for the most complex problems suggests an advantage over students with average FI in maintaining interim results (see also Dehaene et al., 2003; Klimesch et al., 2006; Klimesch et al., 2007). The greater parieto-occipital lower-1 alpha ERD in students with average compared to high FI suggests greater demands on number updating (see also Montojo & Courtney, 2008) for all problems (for further discussion, see 7.4.1 and 7.4.2).

7 General Discussion

7.1 Summary of Results

Three studies were conducted to specify cognitive processes and neural mechanisms that differ as a function of FI and might underlie differences in problem-solving abilities across different mathematical subdivisions between students with average and high FI. The main findings of these studies are: first, results on RTs and error rates support the view of high FI facilitating problem-solving particularly for new tasks. Students with high compared to average FI showed generally better performance on **unfamiliar** geometric analogy tasks but only on complex **familiar** arithmetic and algebraic problems.

Second, eye movements suggest a more adaptive and successful strategy choice in students with high compared to average FI. For solving geometric analogy tasks, students with high FI changed the strategy more often with increasing task difficulty than students with average FI. Strategy changes in students with average but not high FI were associated with a drop in performance.

Third, by analyzing the ERD/ERS in different frequency bands we could identify and specify FI-related differences in the cognitive processes involved in problem-solving and their execution. Specifically, students with high compared to average FI payed more attention on the processing of novel geometric analogy tasks (lower-2 alpha). In familiar arithmetic and algebraic tasks, they maintained interim results for complex problems better than students with average FI (upper alpha), whereas for students with average FI, there were generally greater demands on updating numbers (lower-1 alpha). Learning led to the formation of expectancies (lower-1 alpha) and to an improvement in performance in analogy tasks across both groups. Further, students with high FI became neurally more efficient in moderate tasks due to learning, whereas for students with average FI neural efficiency decreased in hard tasks (alpha band). Students with average and high FI differed in the neural efficiency of strategy execution (lateralization in the alpha band) when controlling for strategy.

In the following, I will discuss these findings on cognitive processes and neural mechanisms and how they might underlie differences in performance that we found for problems from several mathematical subdivisions. First, I will describe the cognitive processes that differ as a function of FI during geometric analogical reasoning since analogies are assumed to play a crucial role for mathematical performance across subdivisions (for review, see Dixon, 2005). I will discuss the relation between FI, analogical reasoning and neural efficiency. Second, I will consider differences in strategy use and comment on the impact of strategy selection on neural efficiency. Third, I will describe the impact of FI on cognitive processes for solving arithmetic and algebraic problems and discuss the role of FI-related differences in analogical reasoning for different mathematical subdivisions. Moreover, I will elaborate on the impact of learning on performance and neural activity during the solving of mathematical problems. Finally, I am going to dwell on some limitations of the present work and put forward suggestions for future research.

7.2 Analogical Reasoning: The Core of Individual Differences in Mathematical Abilities

Hofstadter (2001) refers to analogies “as the core of cognition”. Mathematical cognition requires analogical reasoning for mapping representations of known problems and operations on the relational representation of the problem at hand (for review, see Dixon, 2005). FI contributes to a better performance on analogical reasoning tasks (Sassenberg et al., 2011; van der Meer et al., 2010). Moreover, Preusse et al. (2011) showed that high compared to average FI is associated with greater activity in parietal and smaller activity in frontal brain regions during analogical reasoning indicating differences dependent on FI in the involved cognitive processes. Since these cognitive processes are relevant for solving different mathematical problems specifying FI-related differences in cognitive processes for analogical reasoning constitutes a promising explanatory approach on the impact of FI on mathematical abilities across different subdivisions.

7.2.1 Relational Encoding in High FI

Study I, using unfamiliar geometric analogy tasks, pointed to FI-related differences in attentional processes. Students with high compared to average FI seem to invest more attentional resources in relational encoding, which we assume is indicated by the greater left-central cortical activity in the lower-2 alpha band. In line with this, successful relational encoding was found to be associated with left-hemispheric activity (Vendetti, Johnson, Lemos, & Bunge, 2015) and relational memory is positively correlated with activity in centrally located regions such as the medial temporal lobe and the hippocampus (Davachi, 2006). Furthermore, the lower alpha band is related to the encoding of information (Doppelmayr, Klimesch, Stadler, Pöllhuber, & Heine, 2002) and there is a positive correlation between recognition and recall of task features encoded in geometric analogy tasks and FI (Bethell-Fox et al., 1984). Interestingly, students with high FI solved moderate and hard tasks but not easy tasks more accurately than students with average FI. This could be explained by the fact that solving moderate or hard tasks required successful relational encoding, probably facilitated by a greater investment of attentional resources, compared to easy tasks (i.e., identical relation) where problems could be sufficiently solved by applying elementary perceptual comparison processes.

We can test this assumption of a higher investment in relational encoding as potential source of superior performance on analogy tasks by correlating the left-central lower-2 alpha ERD (i.e., investment in relational encoding) with error rates while controlling for FI-group. These correlations indicate a negative association between the investment in relational encoding for hard tasks and error rates for moderate ($r_{FI\text{-}group} = -.34$, $p < 01$) and hard tasks ($r_{FI\text{-}group} = -.26$, $p < 05$). Thus, those students that encode representations of high relational complexity can solve tasks of such complexity more accurately. The association between high FI and the investment in relational encoding revealed is in line with earlier studies on age-related changes in the ability to integrate relations, which requires the simultaneous representation of relations and inhibition of irrelevant information, relations and prepotent responses (Waltz et al., 1999). This ability improves over childhood and adolescence (Wright, Matlen, Baym, Ferrer, & Bunge, 2007) and decreases over adulthood (Viskontas, Morrison, Holyoak, Hummel, & Knowlton, 2004), while FI in parallel develops over childhood and declines over adulthood (Li et al., 2004). An explanation for a superior relational integration with increasing FI might be that high FI is associated with the ability to efficiently process information (Haier et al., 1988), which will be discussed in the following section.

7.2.2 Neural Efficiency and Analogical Reasoning

Individuals with high compared to average FI solve **familiar** geometric analogy tasks more efficiently with smaller activity in the PFC indicating a lower reliance on executive and integration processes (Preusse et al., 2011). PFC functioning is highly related to the integration of relations while maintaining stimulus representations during upcoming interfering information (for review, see Kane & Engle, 2002; Waltz et al., 1999). By contrast, Study I does not support the view of an efficient processing of information in individuals with high FI for **unfamiliar** geometric analogy tasks as in hard tasks right-hemispheric activity in the broad alpha band, an indicator of general cortical activity (Klimesch, 1999; Pfurtscheller & Lopes da Silva, 1999), was greater for students with high compared to average FI. This is in accordance with studies showing that neural efficiency occurs mostly in familiar but not unfamiliar problems (for review, see Neubauer & Fink, 2009a) and also with the stronger recruitment of parieto-occipital regions in individuals with high compared to average FI which Preusse et al. (2011) report.

Based on the greater right-hemispheric cortical activity for students with high compared to average FI, we assume differences in the visual processing of relational information dependent on FI. The greater investment in relational encoding, we identified for students with high compared to average FI, implies the consideration of more information and reliance on extensive visuospatial processing. In line with this, the lower right-hemispheric cortical activity (i.e., alpha ERD) for students with average compared to high FI during hard tasks probably reflects mental overload due to high visual WM demands. Spatial abilities are associated with right-hemispheric activity (Vogel, Bowers, & Vogel, 2003) and cortical activity, indicated by a smaller alpha ERD or an alpha ERS, decreases in visual WM tasks when individuals experience mental overload (Bashivan, Bidelman, & Yeasin, 2013; Krause et al., 2000). The high error rates in Study I (25.5%) in hard tasks for students with average FI supports the assumption of mental overload, which also replicates previous findings (van der Meer et al., 2010). Moreover, FI-related differences in the processing of visuospatial information is in accordance with a more distributed inspection of RAPM matrices for individuals with high compared to average FI, indicating the attempt to consider and integrate the entire visuospatial information of the tasks (Vigneau et al., 2006). Further, also math-gifted individuals are characterized by extensive visuospatial processing, especially in the most difficult tasks of the RAPM requiring the integration of relations (Desco et al., 2011).

While high compared to average FI goes along with the representation of more information, the inhibition of irrelevant information as a further aspect during relational integration seems not to be increased for high FI. In Study I, the greater cortical activity in students with high compared to average FI over frontal brain regions was indicated by a smaller alpha ERS. According to Klimesch et al. (2007), a smaller frontal alpha ERS reflects less inhibitory top-down control in visuospatial WM tasks. Less inhibitory activity also corresponds to the smaller activity in the PFC reported by Preusse et al. (2011) for students with high compared to average FI. Further, the negative correlation between error rates and the investment in relational encoding (i.e., lower-2 alpha) in Study I remains unchanged when controlling for WM capacity, measured with the Symmetry Span Task by Kane et al. (2004) and the Operation Span Task by Turner and Engle (1989). Such measures of WM capacity imply the maintenance of information in interference-rich contexts requiring inhibitory control. In line with a greater importance of the amount of information that can be considered compared to the inhibition of irrelevant information characterizing high FI, Mogle, Lovett, Stawski, and Sliwinski (2008) emphasize the strong predictive value of secondary memory abilities on FI, whereas WM capacity provides no additional predictive value. Secondary memory is a system with less restricted capacity, in which information can be encoded and maintained with contextual cues for retrieval while other information can be processed (Unsworth & Engle, 2007). However, evidence on the impact of secondary memory on differences in FI is inconsistent (Shelton, Elliott, Matthews, Hill, & Gouvier, 2010).

In sum, greater neural efficiency in individuals with high FI is not a sufficient explanation of differences in the relational processing of information during analogical reasoning. By contrast, individuals with high compared to average FI spent more effort on the processing of spatial information and have an advantage in the integration of relations that allows them to deal with such a large amount of information. This might result from a different information representation, like a stronger reliance on mental imagery during the identification of relations (Sassenberg et al., 2011). According to theories of embodied cognition, which assume grounded cognition and situated action to underlie cognition (Barsalou, 2008) and have been used as theoretical approach in mathematical cognition (Númez, 2004; Núñez & Lakoff, 2005), mental imagery is a simulated action that is planned but not executed. A different processing of relational representations dependent on FI might correspond to a system conceived by O'Boyle et al. (2005) "that may in-part be math (or at least visuospatially) specific, one that highlights the use of imagery based memory representations, which are particularly useful for encoding mathematical concepts and applying them to high-level mathematical reasoning and thinking" (p. 586). It might be also related to differences dependent on FI in strategy use revealed by several studies on analogical reasoning and, thus, affect performance. FI-related differences in strategy use and the processing of tasks from other mathematical subdivisions will be subject of the following two sections.

7.3 Strategy Use in Analogical Reasoning

Several studies using geometric analogy tasks showed that the higher FI is the more often individuals employ the more effective strategy constructive matching (Bethell-Fox et al., 1984; Ullwer et al., 2009; Vigneau et al., 2006). Since constructive matching is characterized by an extensive planning phase, in which a lot of information must be processed to identify relations, differences in the ability to build complex relational representations might account for this finding. The relational complexity is especially high in more difficult tasks, which corresponds to the finding of Bethell-Fox et al. (1984) on a strategic shift from constructive matching to response elimination in more difficult tasks but only for individuals with lower FI-scores. In the present work, we evaluated FI-related differences in strategy use in more detail as I describe below.

7.3.1 Adaptive Strategy Choice in High FI

In Study II, students with high compared to average FI did not use constructive matching more often. Moreover, they changed their strategy from constructive matching to response elimination in moderate and hard tasks (i.e., in response to increasing task demands). Students with average FI changed their strategy only in hard tasks. We suppose that our finding is specific for exceptional high FI. Bethell-Fox et al. (1984) tested individuals from a representative sample of the entire FI spectrum. Vigneau et al. (2006) did not observe strategic shifts in a more selective sample of university students from various disciplines. The students with high FI in the present sample can be assumed to exhibit even higher FI scores.

We propose that high FI is associated with an adaptive strategy choice (see Siegler & Shipley, 1995) across different mathematical subdivisions. On the contrary, average FI is related to the perseveration on sophisticated but sometimes inappropriate strategies and low FI with the employment of less effective strategies especially for difficult tasks. Accordingly, strategy choices in Study II were associated with optimized performance for students with high FI. While error rates increased with increasing task difficulty for students with average FI, we did not observe a drop in performance for students with high FI in response to greater task demands. This is in line with a more adequate strategy use found for highly compared to averagely intelligent children also in other mathematical subdivisions (Hoard et al., 2008). In the present work, students with average FI employed constructive matching in moderate tasks less accurately than students with high FI. Though we also found constructive matching to be the more effective strategy (shorter RT, fewer errors) compared to response elimination (see also Landgraf et al., 2011), one has to note that constructive matching is associated with high WM demands, especially in more difficult tasks (Loesche et al., 2015) and, thus, sometimes inappropriate as we showed.

High FI has been associated with a flexibility that enables individuals to adapt to a variety of different demands (Carpenter et al., 1990). However, it remains unclear, which factors contribute to FI-related differences in the flexibility of strategy selection. A superior selection based on the behavioral efficiency (i.e., fast and accurate) of strategy execution (Siegler & Shipley, 1995) might account for FI-related differences in performance. But how do individuals with high FI know, which strategy is the most efficient under current task demands, especially when the task is new for them? In the present work, we considered further parameters of strategy execution, which might affect individuals' strategy selection and, thus, task performance.

7.3.2 Strategy Execution and High FI

In Study II, we found differences in the strategy execution between students. We found the longer planning phase in constructive matching compared to response elimination could be supported for students with average and high FI (e.g., a longer scan path length before the first midline crossing for constructive matching compared to response elimination). However, students with high FI exhibited a greater processing depth than students with average FI for both strategies, according to Joos, Rötting, and Velichkovsky (2003), indicated by longer mean fixation durations. This corresponds to the findings of Vigneau et al. (2006) on FI-related differences in the amount of information considered for solving geometric analogy tasks and the greater investment in relational encoding we identified for students with high compared to average FI in Study I.

Furthermore, to our best knowledge, we conducted the first study identifying strategies trial-wise to control for differences in strategy use and, thus, to evaluate FI-related differences in neural efficiency for executing strategies. Students with high FI relied more strongly on spatial abilities for strategy execution (right-lateralization; see also Vogel et al., 2003), whereas students with average FI showed a more restricted processing of spatial information and possibly recruited assisting brain regions to master high WM demands (left-lateralization; see also Reuter-Lorenz, Stanczak, & Miller, 1999; Smith, Jonides, & Koeppe, 1996), which is further supported by the findings of increased error rates. These results correspond to the effect of general intelligence on the strategy execution Sanfratello et al. (2014) report for visual WM tasks. Although their findings only apply to the tendency of using a specific strategy as the intraindividual variability in strategy use was not considered. An intense processing of spatial features even for the execution of individual strategies might imply that individuals with high FI select strategies, for which they are capable to consider all information provided by the task (see also Vigneau et al., 2006). This is a reasonable approach for solving unfamiliar tasks since it avoids a bias towards single task features and allows for switching to a less demanding but more appropriate strategy when task demands increase. Further studies on the role of neural efficiency of strategy execution for adaptive strategy choices are needed.

Besides our interest in differences in strategy execution, we combined EEG and eye movement measures to distinguish between differences in neural activity stemming from the selection of different strategies and those due to differences in the execution of the same strategies. Thereby, we addressed a critique by Poldrack (2015) on earlier studies investigating individual differences in neural efficiency that assume individuals to perform the same computations for problem-solving. Unlike these studies, we tested this assumption (see 7.3.1) and controlled for differences in strategy use before evaluating FI-related differences in neural efficiency. In the following, I will discuss the impact of strategy selection on differences in neural efficiency.

7.3.3 Strategy Selection and Its Impact on Individual Differences in Neural Efficiency

The comparison between FI-related differences in neural efficiency in Study I, where differences in strategy use were not considered, and those in Study II indicates that differences in neural efficiency can be partly ascribed to differences in strategy selection. We directly showed that differences in right-hemispheric cortical activity in Study I did not occur after controlling for strategy in Study II, except for differences in lateralization. Accordingly, findings of Grabner et al. (2007) on arithmetic problem-solving indirectly point to differences in brain activation resulting from the use of different strategies dependent on mathematical abilities. Furthermore, strategy selection has already been shown to explain differences in cortical activity dependent on task difficulty (Grabner & De Smedt, 2011). In Study II, we only included a subsample of Study I, which may underlie the different findings of the studies. To unequivocally evaluate the impact of strategy selection on differences in neural efficiency, we need to prove for Study II whether FI-related differences in neural efficiency occur when we do not control for differences in strategy use and that these potential differences vanish when we control for strategy.

Without controlling for strategy, we find FI-related differences in the cortical activity corresponding to differences in the investment in relational encoding based on the processing of more spatial information by individuals with high compared to average FI. Unlike students with high FI, those with average FI probably solved easy tasks by elementary perceptual comparison processes as the right-hemispheric cortical activity for easy tasks was smaller compared to difficult tasks and compared to students with high FI. The effort hypothesis proposes a positive correlation between FI and cortical activity even in easy tasks (Ahern & Beatty, 1979) and is supported for several mathematical subdivisions (Dix & van der Meer, 2015; Haier, Siegel, Tang, et al., 1992; Larson, Haier, LaCasse, & Hazen, 1995), especially under time pressure (Lamm et al., 2001). For hard tasks, students with average FI recruited assisting brain regions to deal with high WM demands indicated by greater left-hemispheric cortical activity for hard compared to easy tasks (see also Smith et al., 1996). Thus, when we do not control for differences in strategy use, Study II suggests like Study I FI-related differences in the spatial processing of task features for some levels of task difficulty (for a discussion on differences between Study I and Study II, see 7.5).

Thus, the present work shows that strategy selection affects FI-related differences in cortical activity. When controlling for strategy selection, Study II points to FI-related differences in the neural efficiency of strategy execution (see 7.3.2). However, for the employment of constructive matching, we could show that this difference in strategy execution is independent of task difficulty, which is opposed to the effect of task difficulty we find when we do not control for strategy. Thus, it seems to be necessary to control for strategy when evaluating differences in the neural efficiency (see also Poldrack, 2015). Further studies need to test whether the effect of strategy selection on neural efficiency and differences in the neural efficiency of strategy execution are generalizable to other mathematical subdivision and problem-solving domains.

In the following, I will elaborate what role a more intense processing of spatial features by students with high compared to average FI plays for solving problems across different mathematical subdivisions, which we even found for the execution of individual strategies in geometric analogy tasks.

7.4 Relational Representations: Explaining the Impact of FI on Mathematical Abilities

Relational reasoning has been shown to affect students' mathematical achievements and the learning of mathematical concepts and operations (Dumas, Alexander, & Grossnickle, 2013). According to Dixon (2005), even for solving familiar mathematical problems one needs to identify the relational structure of the problem to map representations of known problems and operations on the representation of this structure. We observed a superior ability of individuals with high compared to average FI to process spatial information and to integrate relations in geometric analogy tasks. We assume that this superiority also underlies differences in other mathematical subdivisions and might account for the reported flexibility associated with high FI, which allows adapting to a variety of different task demands (Carpenter et al., 1990). One shortcoming of earlier studies on mathematical abilities is that they normally focus on one single subdivision

(Floyd et al., 2003). In the present work, we considered, besides FI-related differences in the solving of geometric tasks, also the processing of arithmetic and algebraic problems.

7.4.1 The Impact of FI on Simplifying Complex Arithmetic and Algebraic Terms

FI-related differences in the performance on familiar arithmetic and algebraic problems are associated with a greater investment in the representation of numerical quantity for students with high compared to average FI (Dix & van der Meer, 2015). Beyond this, findings of Study III suggest differences dependent on FI for memory storage/updating. We assume that students with average FI were confronted with greater demands on number updating than their highly intelligent peers indicated by a greater parieto-occipital ERD in the lower-1 alpha band for students with average compared to high FI. For mental arithmetic, parietal brain regions get activated during the updating of numbers (Montojo & Courtney, 2008) and memory-updating abilities are associated with better performance in arithmetic (Passolunghi & Pazzaglia, 2004).

Since memory-updating abilities correlate positively with FI (Friedman et al., 2008) the FI-related differences in demands on number updating might reflect differences in updating abilities. However, Study III suggests that differences stem from the use of different strategies involving number updating to a different extent. Students with high compared to average FI probably relied more strongly on fact retrieval as a trend (p = .09) toward a stronger left-hemispheric theta ERS indicates (see also De Smedt et al., 2009; Grabner & De Smedt, 2011). Earlier studies also reported that individuals with high compared to average FI employ fact retrieval more often (Geary & Brown, 1991) and more successfully (Hoard et al., 2008), which is reasonable because of the positive relation between FI and crystallized intelligence, the ability to use acquired knowledge (Beauducel, Brocke, & Liepmann, 2001; Beauducel & Kersting, 2002). By contrast, students with average FI might employ multi-step procedures with greater demands on number updating.

According to findings on an adaptive strategy choice for high FI (Hoard et al., 2008), the use of more appropriate strategies might explain differences in performance. In familiar tasks (Study III), we showed strategy choices to be crucial for solving complex problems requiring additional memory storage/updating and executive processes. Students with high compared to average FI were faster only for the complex problems, that is, simplification of advanced algebraic expressions and complex algebraic expressions. For the latter, students with high FI had an advantage in retrieving interim results, which we assume based on the greater left-hemispheric cortical activity in the upper alpha band found for students with high compared to average FI. In memory tasks, a great upper alpha ERD occurs during retrieval (Klimesch et al., 2006; Klimesch et al., 2007) and, for mathematics, activity in the left angular gyrus is associated with the retrieval of interim results (Dehaene et al., 2003) and high mathematical abilities (Grabner et al., 2007). Supporting this assumption, students with average FI reported difficulties in maintaining arithmetic interim results when simplifying complex algebraic expressions, which caused them to recalculate the problems and corresponds to the longer RTs compared to students with high FI.

Contrary to the findings of Study III, FI played a minor role for the performance and cortical activity (upper alpha) in an earlier study on familiar memory-based reasoning tasks (Grabner, Stern, & Neubauer, 2003). However, we assume that the complex problems in the present work were especially demanding since we combined two domains, namely arithmetic and algebra. Greater demands on the updating of numbers for students with average compared to high FI occurred for simple and complex problems across subdivisions, probably resulting from differences in strategy use. Study III suggests that maintaining interim results from one subdivision while updating numbers during multi-step procedures in the other is especially challenging. Interfering information might have caused students with average FI to forget arithmetic interim results. Operand intrusion (e.g., $8 \times 4 = 24$), for instance, causes arithmetical errors (Campbell, 1994). However, it remains open why students with average FI needed more steps for solving problems that mainly require the execution of well-known routines. All participants in the present work probably had wide mathematical knowledge due to their special background. One possible explanation is that students differ in their ability to identify the relational structure of problems and, thus, in their ability to map and retrieve routines that match this structure (Dixon, 2005). This is in line with the findings of Study I on FI-related differences in relational encoding during analogical reasoning and will be examined in the following section.

7.4.2 Relational Representations in Arithmetic and Algebraic Problem-Solving

For geometric analogy tasks, Study I revealed that students with high FI build relational representations based on an intense processing of spatial information, which was associated with better performance compared to students with average FI. We propose that such greater investment also occurred for solving arithmetic and algebraic problems and, thus, helped individuals with high FI identifying the problems' relational structure to map known problems and operations on it (Dixon, 2005). Consistently, Study III revealed smaller demands on the updating of numbers for students with high compared to average FI, probably resulting from the use of well-known routines (i.e., fact retrieval). Based on the hypothesis that differences in relational encoding might underlie FI-related differences in mathematical cognition across different subdivisions allowing for the use of less demanding strategies, we proved that the investment in relational encoding in Study I on geometric analogy tasks, is negatively correlated with demands on number updating in Study III on arithmetic and algebraic problem-solving.

We ran a post-hoc linear regression analysis with the parietal lower-1 alpha ERD in Study III (i.e., number updating) as dependent variable and the left-central lower-2 alpha ERD in Study I (i.e., relational encoding) as predictor variable. For this, we had to take into account that the two lower alpha bands are not independent, but respond similarly to cognitive load. Thus, we included in a first regression model ($R^2_{adjusted} = .65$; $p < .001$) all measures of the lower-2 alpha ERD in Study III (i.e., unspecific lower-2 alpha activity), and in a second model ($R^2_{adjusted} = .83$; $p < .05$) additionally all measures of the lower-1 alpha ERD in Study I (i.e., unspecific lower-1 alpha activity) that suppressed the assumed negative correlation. FI-group as additional predictor variable did not contribute significantly to the model and was not further considered. A third model ($R^2_{adjusted} = .88$; $p < .05$) additionally including the left-central lower-2 alpha ERD in Study I (i.e., relational encoding) for hard tasks revealed the negative association ($\beta = -.26$) with the parietal lower-1 alpha ERD in Study III (i.e., number updating). The analysis was confined to activity during hard tasks since relational complexity was highest for these tasks and the functional dissociation (expressed in a decreased positive correlation) between the alpha sub-bands increases with increasing task demands (A. Fink, Grabner, Neuper, & Neubauer, 2005).

This negative relation is a first hint to the crucial role the ability to build representations of high relational complexity in individuals with high FI might play in explaining difference in mathematical abilities. It supports the assumption made by an earlier study that students with high compared to average FI encode numerical quantity rather in the visual system of the intraparietal sulcus (Dehaene et al., 2003) for solving arithmetic and algebraic problems (Dix & van der Meer, 2015). This greater reliance on the visual system in mathematical cognition seems to be specific for high FI but not for measures of crystallized intelligence (Ahern & Beatty, 1979; Dix & van der Meer, 2015; G. Lee, Ojha, Kang, & Lee, 2015). Furthermore, it corresponds to the proposal by O'Boyle et al. (2005) that a visuospatial system is especially useful for solving mathematical problems and connects this system with FI-related differences in the processing of visuospatial information and mathematical abilities. The present work indicates that a stronger reliance on this visuospatial system allows individuals with high compared to average FI to use WM resources more optimally, which I further discuss in the following referring to findings on the relation between FI and learning-related changes in WM demands.

7.4.3 The Impact of Learning on the Building of Relational Representations

The difficulties of students with average FI, particularly for solving complex familiar and unfamiliar problems revealed in the present work, indicate that WM capacity plays a crucial role for explaining FI-related differences in the ability to build representations of high relational complexity. Study I suggests that a learning-related reduction of WM demands allows even individuals with lower abilities in relational integration to solve tasks successfully. Accordingly, RTs for moderate and hard tasks and error rates for hard tasks decreased from the first half to the second half of the experiment for all students. Moreover, post-hoc we find the investment in relational encoding (lower-2 alpha) to correlate with error rates only in the first half of the experiment but not in the second half. WM capacity is positively correlated with FI (Kyllonen & Christal, 1990) and learning has been shown to reduce WM demands (Schoenfeld, 1992), though it is controversial whether this reduction results from the use of different strategies or from the automatization of involved cognitive processes (Tronsky, 2005).

We showed that learning contributed to the formation of expectancies that directed participant's visual attention to task-relevant features probably facilitating relational integration. This is indicated by an increase of right-hemispheric cortical activity in the lower-1 alpha band over the course of the experiment. In line with this, individuals can develop a memory for a visual context (e.g., the structure of geometric patterns) that directs their attention (Chun & Jiang, 1998) and the lower-1 alpha band is associated with expectancy driven alertness as one aspect of attention (Klimesch et al., 1998). Alertness is related to activity in right-hemispheric brain regions (Sturm & Willmes, 2001) and alertness training leads to an increase of this activity (Pizzamiglio et al., 1998; Sturm, 2004). Stimulating right-hemispheric regions disrupts alpha ERD modulating spatial attention and thus impairing performance (Capotosto, Babiloni, Romani, & Corbetta, 2009). A focused visual attention on task-relevant features as result of top-down expectations due to learning has been already described by Grossberg (1999). Further, it might facilitate relational integration, which shares WM resources with the resolution of interfering information (Cho, Holyoak, & Cannon, 2007).

Findings of Study I highlight that learning can compensate for ability differences but also underpin the assumption of FI-related differences in the relational representation of information. Neural efficiency increased (i.e. decrease of the right-hemispheric ERD in the broad alpha band) in students with high FI for moderate tasks but decreased in students with average FI for hard tasks. Though, this is in accordance with the assumption of a stronger learning-related increase in neural efficiency with increasing FI (Haier, Siegel, Tang, et al., 1992; Neubauer et al., 2004), the decrease in neural efficiency (i.e. increase of the right-hemispheric ERD in the broad alpha band) for students with average FI seems rather to reflect the overcoming of mental overload due to the reduction of WM demands by focusing on task-relevant features. Accordingly, cortical activity in hard tasks did not differ dependent on FI in the second half of the experiment. Consistently, Grabner et al. (2009) report that activity in the angular gyrus differs dependent on mathematical abilities only in untrained but not in trained arithmetic tasks. Learning-related reductions of WM demands did not result in an increase in neural efficiency for students with high FI in hard tasks. Several studies showed that some participants are not affected by a secondary task changing WM demands, which indicates that participants engage in different processes for solving the primary task (Kane & Engle, 2000; Rosen & Engle, 1997). Thus, the lacking impact of learning on neural efficiency in students with high FI suggests that these students might rely on different processes for representing and integrating relations, for instance, on mental imagery (Sassenberg et al., 2011). Future research need to specify these processes further and also should address the limitations of the present work outlined below.

7.5 Limitations of the Present Work

The sample size of Study II allowed detecting FI-related differences in error rates but not in RTs for solving the tasks. In Study I, students with high compared to average FI solved tasks faster and more accurately. Thus, FI might affect the accuracy of problem-solving more strongly than the speed. This is supported by the lower effect sizes for RTs compared to error rates in Study I and also in a study by van der Meer et al. (2010) using a similar unfamiliar geometric analogy task. Based on the effect sizes of this latter study, the sample of Study II but not of Study I was too small to detect an effect of FI on RTs. According to Faul, Erdfelder, Lang, and Buchner (2007), a sample size of $N = 66$, with $\alpha=.05$ and $1-\beta = .95$, would have been recommended.

The smaller and, thus, different sample in Study II but also differences in the data processing might explain why in this study cortical activity in the broad alpha band was greater for students with high compared to average FI in **easy** tasks whereas in Study I FI-related differences occurred in **hard** tasks. In Study II, trials that we could not ascribe unambiguously to constructive matching or response elimination (i.e., trials with two midline crossings; 16%) were not considered and, thus, processes such as the choice which of the pattern pairs serves as source pair might not have been covered. Further, Study I distinguished between the first and second half of the experiment. There were no FI-related differences in cortical activity in hard tasks over the total time course of the experiment, which is in accordance with findings of Study II. Moreover, splitting the trial number in Study I increased the variance in the conditions (first versus second half) for easy tasks. The easy conditions only comprised trials with identical relations whereas moderate and hard tasks comprised two types of relation each. A lower number of easy task trials might be linked with a worse signal-to-noise ratio in Study I and with a masking of the effect of FI.

In Study II, we could not evaluate FI-related differences in neural efficiency of executing response elimination dependent on task difficulty as 17 students did not use response elimination for all conditions. Thus, we also could not include task difficulty in the analysis for comparing cortical activity for constructive matching and response elimination. This might explain why we did not find differences in cortical activity between the two strategies. Such differences might only occur in difficult tasks (Bethell-Fox et al., 1984). Despite the consideration of a greater sample for further experiments, some new research questions arose from the presented findings. I want to conclude this work by proposing several ideas how future studies can address these questions and with some final remarks at the end.

7.6 Directions for Future Research and Practical Implications

According to the adaptive strategy choice model (Siegler & Shipley, 1995), individuals select strategies that can be executed efficiently (i.e., fast and accurate). It is not known whether a neurally efficient execution affects strategy selection, too. We conducted the first study evaluating the neural efficiency of strategy execution and showed that students with average FI recruit assisting left-hemispheric regions, which might explain a different strategy selection. The impact of the efficient execution of a strategy on its selection could be studied in different ways. An experiment could consist of different blocks, forced and non-forced strategy choice blocks. Strategy selection in the non-forced choice blocks should depend on the neural efficiency of the strategy execution in the forced-choice blocks. Strategies that can be executed with a greater neural efficiency are assumed to be selected more frequently. To avoid influences of strategy instruction in the forced-choice blocks on the strategy selection in the non-forced choice blocks one could also stimulate task-relevant brain regions using tanscranial Direct Current Stimulation to improve or diminish their functioning (Hauser, Rotzer, Grabner, Merillat, & Jancke, 2013; Meinzer, Lindenberg, Antonenko, Flaisch, & Floel, 2013; Zheng, Alsop, & Schlaug, 2011).

We showed that high FI allows building relational representations based on an intense processing of spatial information and assume that individuals engage in different processes dependent on FI to represent and integrate relations (e.g., mental imagery; Sassenberg et al., 2011). To further specify FI-related differences in the relational integration one could present only one pattern pair and analyze the neural dynamics underlying the identification of the relation between the patterns. For EEG measures, the most important strength is the high temporal resolution – to the detriment of the spatial precision. Often, algorithms are not able to achieve a high temporal resolution while distinguishing precisely between different bands of the EEG. Recently, Ivanova and Kutin (2015) adapted the Hilbert-Huang-Transform (Huang & Shen, 2005) for multichannel analyses, which allows exploring cognitive processes in time and frequency with high resolution.

Since the integration of information requires unbinding relations, the superiority of students with high FI to build representations of high relational complexity might result from their ability to unbind features and to update episodic representations (Colzato, van Wouwe, Lavender, &

Hommel, 2006). One can explore the role of feature unbinding for the identification of relations by comparing performance, eye movements (indicating the information search) and cortical activity (indicating the underlying neural mechanisms), for trials preceded by trials of the same type of relation or by trials of a different type of relation. Individuals with high FI should be less negatively affected by a mismatch with the relation of the preceding trial than students with average FI (i.e., better performance, fewer fixations on task-irrelevant areas, less decreased cortical activity in the lower-2 alpha band). Since for analogy tasks distractors display two different relations, they are not useful to determine the role of feature unbinding. However, one can present only one pair of the patterns used in the present work, which are also suitable to determine the impact of the visual context on feature binging/unbinding since we can vary the type of relation while using the same pattern or vary the pattern while using the same type of relation.

Unlike earlier studies, the present work considered several mathematical subdivisions and suggests that the ability to build representations of high relational complexity explains FI-related differences in mathematical performance across different subdivisions. Highly trained routines can only be applied if the relational structure of the problem can be identified (Dixon, 2005). Future studies could specify FI-related differences in the information used to represent the structure of arithmetic and algebraic problems using eye movement analyses (see also H. J. Green et al., 2007). In addition, one can manipulate the task presentation and, thus, the difficulty to identify relations. The task presentation affects internal representations and related performance (E. Fink, 2002; Koedinger, Alibali, & Nathan, 2008). If we manipulate the position of elements in complex expression this should facilitate or hamper the identification of relations between elements. For instance, canceling down numbers might occur more often, when numbers are presented one above the other instead of on opposite sides of the fraction. We assume that FI-related differences in the selection of fact retrieval are smaller in the first than in the second example.

Accordingly, we tested for geometric analogy tasks whether manipulating the presentation of the patterns helps to identify relations. We presented the same patterns used in the present work, but only one pair. Participants were instructed to identify the relation between the patterns that appeared either next to or above each other. According to Sassenberg et al. (2011), we assumed participants to simulate the mirroring to identify relations by mentally rotating the patterns. Findings revealed better performance in trials were the rotation direction of the relation corresponded to the presentation of the patterns (congruent trials; e.g., vertical relation for patterns presented next to each other or horizontal relation for patterns presented above each other). Pupil dilation, the aggregate measure of mental activity (Beatty & Lucero-Wagoner, 2000), was smaller in congruent trials indicating lower demands compared to incongruent trials. High FI was associated with better performance but only in incongruent trials. Thus, task presentation helped to identify relations leading to an improved performance and the reduction of FI-related differences.

The internal task representation can also be changed by learning new problem-solving strategies (Alibali, Phillips, & Fischer, 2009). We propose that instructing appropriate strategies facilitating the identification of relations is a promising approach in education to improve performance and reduce individual differences. However, one needs to select instructions on strategies carefully. For instance, Loesche et al. (2015) instructed participants on the rules for solving the RAPM test. When individuals knew the rules, they employed the more effective strategy constructive matching more often. However, the correlation between task performance and individuals' WM capacity increased simultaneously. Thus, such instructions that probably do not affect the task representation but only encourage the choice of more demanding strategies should not be the first choice when the support of individuals with lower FI, which have lower WM capacities (Kyllonen & Christal, 1990), is intended. Passolunghi and Pazzaglia (2004) discussed that writing down interim results for arithmetic problems might reduce WM demands and, thus, improve performance. However, they showed that performing the current calculation step still relied on updating abilities, which are positively correlated with FI (Friedman et al., 2008). Thus, to determine appropriate instructions in education further research should specify the role of WM abilities for explaining FI-related differences in mathematical performance, for instance, by using secondary tasks draining WM resources.

7.7 Conclusions

In conclusion, the present work specified FI-related differences in the cognitive processes and neural mechanisms that underlie differences in performance on problems across different mathematical subdivisions (geometry, arithmetic, algebra). Using a multi-methodological approach including behavioral data, eye movements and the ERD/ERS in the theta and alpha band we shed light on the complex interplay between FI and mathematical abilities. We showed that high FI is associated with the ability to build relational representations based on an intense processing of spatial information facilitating the solving of geometric analogy tasks. This was reflected in increased cortical activity. Thus, neural efficiency, one feature of high FI, is not an appropriate concept for explaining FI-related differences in mathematical cognition. We identified an adaptive strategy choice as one characteristic of high FI generalizing earlier findings to another mathematical subdivision. Further, we conducted the first study identifying strategies trial-wise to evaluate FI-related differences in the neural efficiency of executing strategies and the impact of strategy selection on differences in cortical activity. Findings point to FI-related differences in the neural efficiency of strategy execution, which might constitute a factor influencing strategy selection. The disappearance of FI-related differences in the amount of cortical activity when controlling for strategy selection highlights the need to consider differences in strategy use before evaluating differences in neural activity. For solving arithmetic and algebraic problems, high FI was associated with low demands on the updating of numbers and, thus, with better performance compared to students with average FI in complex tasks with high WM demands. An advantage to identify the relational structure of problems probably allowed mapping and retrieving less demanding routines (i.e., fact retrieval) that match this structure. Accordingly, the investment in relational encoding during geometric analogical reasoning was negatively correlated with the demands of updating numbers in arithmetic and algebra. Thus, the ability to build representations of high relational complexity might be a key aspect explaining FI-related difference in mathematical abilities. Individuals seem to engage in different processes for relational integration dependent on FI (e.g., mental imagery), supported by FI-related differences found in the learning-induced reduction of WM demands by focusing on task-relevant features. Future studies need to identify the mechanisms underlying a superior ability to build representations of high relational complexity, for instance, considering FI-related differences in the ability to unbind relations of episodic representations. Manipulating the task presentation could facilitate the identification of relations and thereby reduce FI-related differences in performance on mathematical problems. Moreover, we need to specify the role of WM abilities for explaining differences in problem-solving dependent on FI, for instance, by using secondary tasks draining WM resources.

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